MATH 6: HANDOUT VII
SETS 1. INTRODUCTION

By the word *set*, we mean any collection of objects: numbers, letters,. . . . Most of the sets we will consider, consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

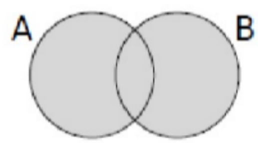
Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

Other notation:
$x \in A$ means "x is in A", or "x is an element of A"
$x \notin A$ means "x is not in A"
$\varnothing$ is the empty set, or set which contains no elements. It is sometimes useful for the same reasons it is useful to have a notation for number 0.

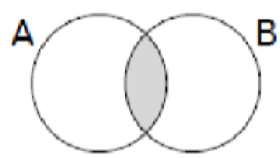
Operations between sets. $A \cup B$: union of A and B. It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



$A \cap B$: intersection of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



$\overline{A}$: complement of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.
$|A|$: number of elements in a set A (if this set is finite).

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: [and] represent that the end point is included, while (and) indicate that the end point is excluded.
$[a, b] = \{x \mid a \le x \le b\}$ is the interval from a to b (including endpoints),
$(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
$[a, \infty) = \{x \mid a \le x\}$ is the half-line from a to infinity (including a),
$(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

HOMEWORK

1. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.
Based on all of this, can you explain why it is impossible that Al comes to the party?

2. Let $A = [1, 3] = \{x \mid 1 \le x \le 3\}$, $B = \{x \mid x \ge 2\}$, $C = \{x \mid x \le 1.5\}$. Describe $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

3. Draw the following sets on the number line:
(a) Set of all numbers x satisfying $x \le 2$ and $x \ge -5$;
(b) Set of all numbers x satisfying $x \le 2$ or $x \ge -5$
(c) Set of all numbers x satisfying $x \le -5$ or $x \ge 2$

4. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?
(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

5. In this problem, we denote by $|A|$ the number of elements in a finite set A.
(a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.
*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$.

6. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer? chess? both? neither?

7. For each of the sets below, draw it on the number line and then describe its complement:
(a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$

8. (AMC) Which of the following has the largest shaded area?