

MATH 6
HANDOUT 6: SETS CONTINUED

COUNTING

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \dots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(see problem 6 below).

PRODUCT RULE

If we need to choose a pair of values, and there are a ways to choose the first value and b ways to choose the second, then there are ab ways to choose the pair.

For example, a position on a chessboard is described by a pair like a4; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 64$ possible positions.

It works similar for triples, quadruples, For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, we get $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences we can get.

1. Let $A = [1, 3] = \{x \mid 1 \leq x \leq 3\}$, $B = \{x \mid x \geq 2\}$, $C = \{x \mid x \leq 1.5\}$. Draw on the number line the following sets: \overline{A} , \overline{B} , \overline{C} , $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.
2. Long ago, in some town a phone number consisted of a letter followed by 3 digits (e.g. K651). How many possible phone numbers could there be in that town?[Note: digits could be zero, so a number like X000 was allowed.]
3. If we roll 3 dice (one red, the other white, and the third one, black), how many combinations are possible? How many combinations in which the sum of values is exactly 4?
4. A **subset** of a set A is a set formed by taking some (possibly all) elements of A ; for example, the set $\{2, 4, 6, 8\}$ is a subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set which contains no elements at all and S itself).
Can you guess the general rule: if set S has n elements, how many subsets does it have?
5. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?
(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
6. In this problem, we denote by $|A|$ the number of elements in a finite set A .
(a) Show that for two sets A, B , we have $|A \cup B| = |A| + |B| - |A \cap B|$.
*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|, |A \cap B|, |A \cap C|, |B \cap C|$.

7. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer? chess? both? neither?
8. 150 people at a Van Halen concert were asked if they knew how to play piano, drums or guitar.
- (a) 18 people could play none of these instruments.
 - (b) 10 people could play all three of these instruments.
 - (c) 77 people could play drums or guitar but could not play piano.
 - (d) 73 people could play guitar.
 - (e) 49 people could play at least two of these instruments.
 - (f) 13 people could play piano and guitar but could not play drums.
 - (g) 21 people could play piano and drums.
- How many people can play piano? drums?
- *9. A barber in a small town decides that he will shave all men who do not shave themselves (and only them). Should he shave himself? [Of course, the barber is a man.]