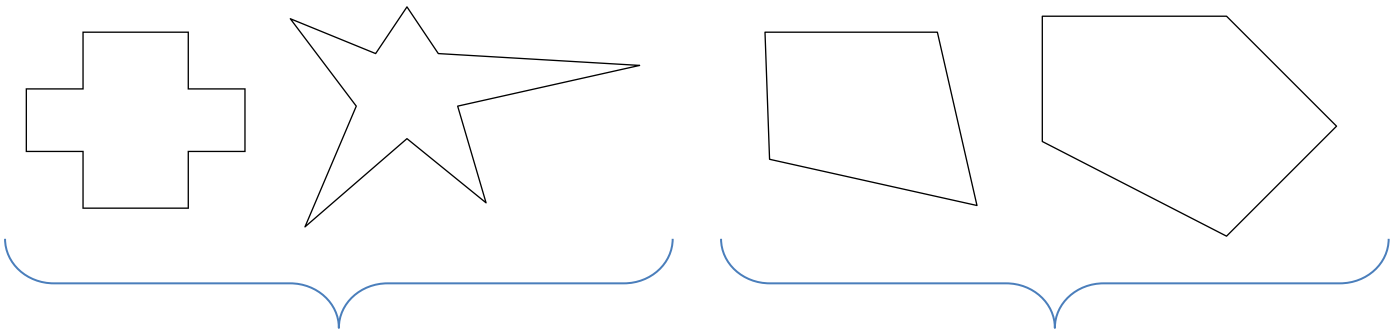
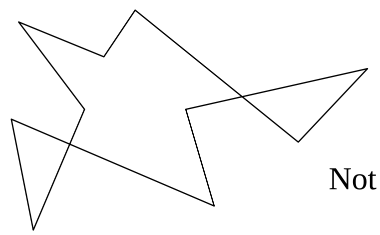
Classwork 19.

The set of line segments, drawn in a way that the beginning of the next segment is the end of the previous one, without any self-crossing (where no segment intersects another segment), and where the end of the last segment is also the beginning of the first segment, forms a polygon.



Concave polygons

Convex polygons

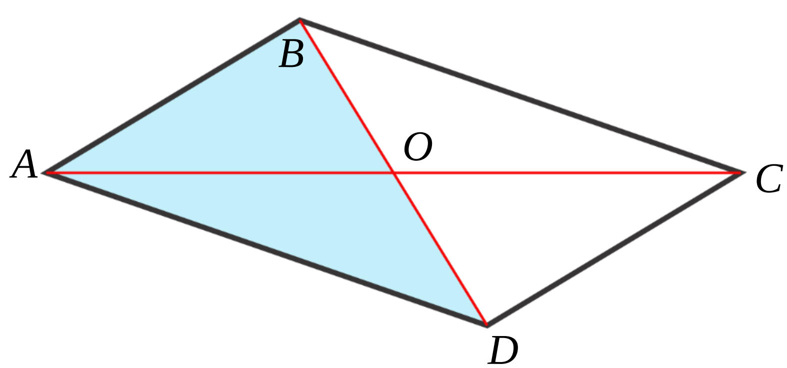


Not a polygon polygons

Simplest polygon is a triangle.

Quadrilaterals are polygons with four sides and four angles. We can classify quadrilaterals into groups with different number of pairs of parallel lines.

- Quadrilaterals with no parallel lines (as one of the polygons above).
- Quadrilaterals with one pair of parallel lines are called trapezoids. (Some mathematicians define trapezoids as a quadrilateral with **at least one** pair of parallel lines, in this case the set of parallelograms is a subset of the set of trapezoids, but this point of view is considered to be not popular among mathematicians).
- Parallelograms are quadrilaterals with two pairs of parallel lines.

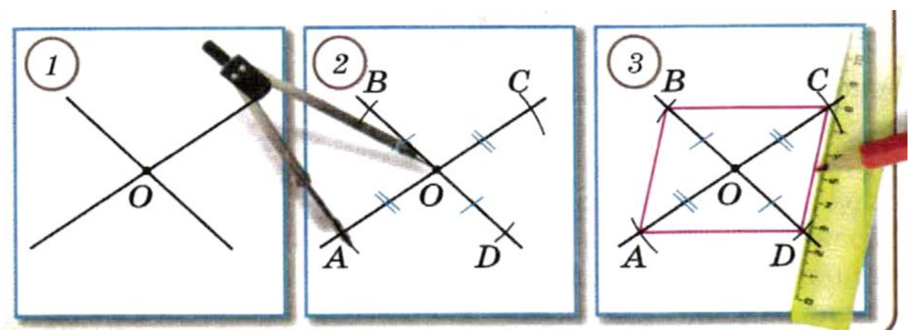


Copy the parallelogram onto a piece of parchment paper, and cut it out. Place the cut shape onto a picture, and align it. Place the needle of a compass on the intersection of the diagonals, and rotate the parchment paper 180 degrees. From this experiment we can find a few properties of parallelograms.

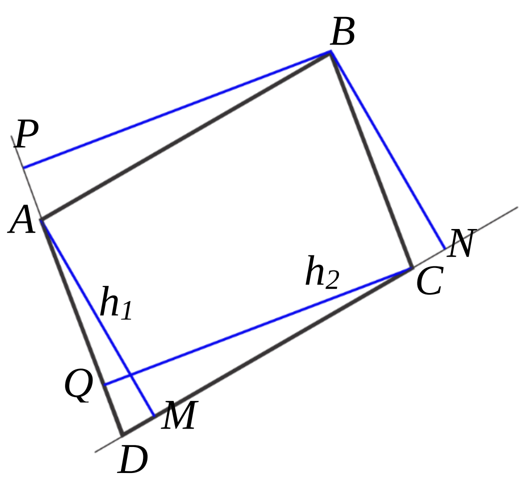
- Their sides not only parallel, but also equal.
- Diagonal divides a parallelogram onto two equal (congruent) triangles.
- Diagonals intersect at the midpoint.

The third property can help us to create algorithm of constructing parallelogram:

1. Draw two intersecting lines.
2. Using compass mark two pairs of equal segments from the point of intersection:
$$[AO] = [OC], [BO] = [OD].$$
3. Connect points A, B, C, D by segments. ABCD is a parallelogram.



How do we call a parallelogram with all right angles? Parallelograms with equal sides are called rhombuses.

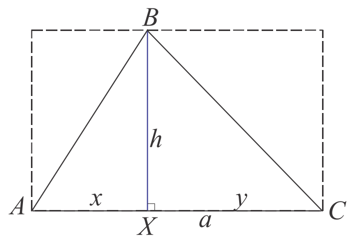


Area of a parallelogram.   On the picture below. ABCD is a parallelogram. Segments [AM] and [BN] are equal and perpendicular to lines (DC) and (AB). Triangles DAM and CBN are equal. You can see it by superimposing them (and it can be proved based on the theorems of triangle equalities). So the area of parallelogram is equale to the area of a rectangle

$$S_{ABNM} = |MN| \cdot h_1 = |DC| \cdot h_1$$

(h_1 is an altitude, distance between a pair of parallel lines. Of course, it’s also equal to

$$S_{ABNM} = |AD| \cdot h_2$$

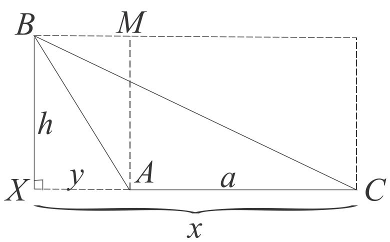


The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

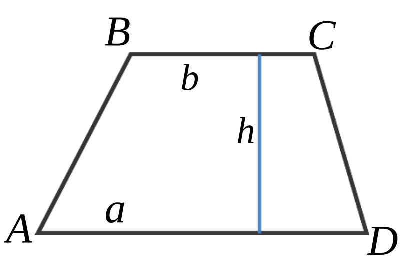
$$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$$

$$S_{\Delta ABX} = \frac{1}{2} h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2} h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$



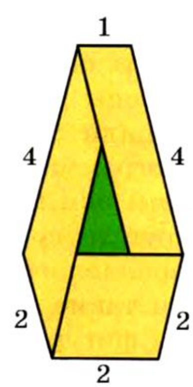
$$S_{\Delta ABC} = \frac{1}{2} h \cdot x + \frac{1}{2} h \cdot y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle? Area of trapezoid?



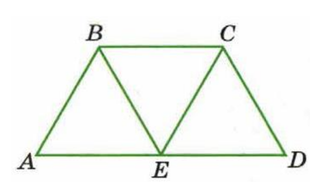
Exercises:

1. Draw two different parallelograms with diagonal, equals to 4 cm and 6 cm.



2. All quadrilaterals on the picture are parallelograms. What are the lengths of the sides of green triangle?

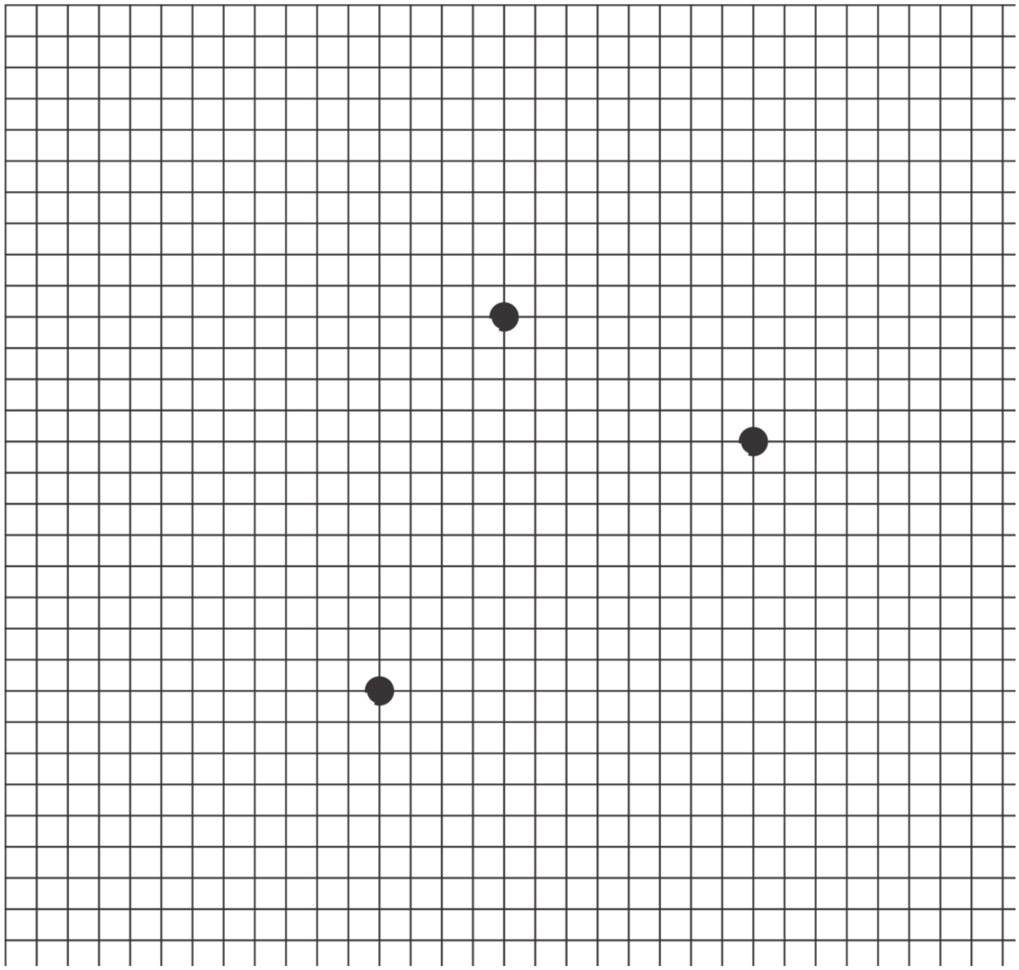
3. Quadrilateral is not a parallelogram, but it has one pair of parallel sides and one pair of equal sides. Draw this quadrilateral.



4. ABCE and BEDC are rhombuses. |BC| is 3 cm. Find the perimeter of triangle BEC. What are the angles of this triangle.

5. Diagonals of a rectangle are equal, diagonals of a square not only equal but also perpendicular. Draw two different rectangles with diagonal 6 cm. Draw a square with diagonal 6 cm. Is it possible to draw another square with a diagonal length of 6 cm, which is not equal to the first one?

6. Points A, B, and C are vertices of a parallelogram. Draw all possible parallelograms.



7. Draw a rhombus with diagonals 4 cm and 6 cm.