Classwork 18.

Collatz conjecture

The **Collatz conjecture** is one of the most famous unsolved problems in mathematics. The conjecture asks whether repeating two simple arithmetic operations will eventually transform every positive integer into 1. It concerns sequences of integers in which each term is obtained from the previous term as follows:

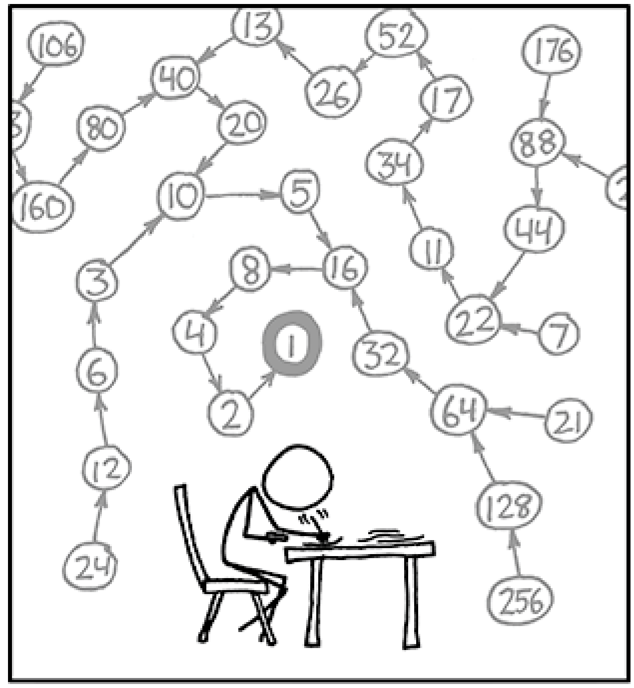
- If the previous term is even, the next term is one half of the previous term.

$$n : 2$$

- If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that these sequences always reach 1, no matter which positive integer is chosen to start the sequence.

$$3n + 1$$

Solving this problem means proving that for any natural number, there exists a finite number of steps (which may be very large), as described above, that reduce the number to one; or proving that there exists at least one number for which this reduction to one is not possible.



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Fermat's Last Theorem (sometimes called **Fermat's conjecture**, especially in older texts) states that no three positive integers *a*, *b*, and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2. The cases $n = 1$ and $n = 2$ have been known since antiquity to have infinitely many solutions.

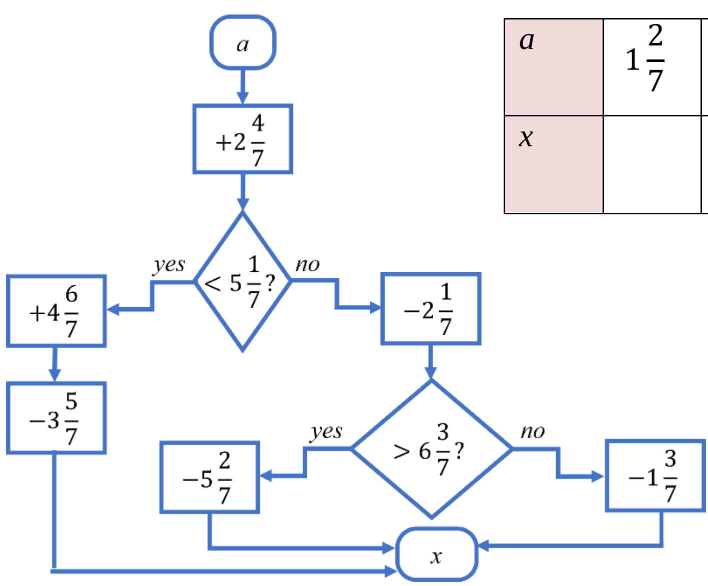
The proposition was first stated as a theorem by Pierre de Fermat around 1637 in the margin of a copy of *Arithmetica*. Fermat added that he had a proof that was too large to fit in the margin. Although other statements claimed by Fermat without proof were subsequently proven by others and credited as theorems of Fermat (for example, Fermat's theorem on sums of two squares), Fermat's Last Theorem resisted proof, leading to doubt that Fermat ever had a correct proof. Consequently the proposition became known as a conjecture rather than a theorem. After 358 years of effort by mathematicians, the first successful proof was released in 1994 by Andrew Wiles and formally published in 1995. It was described as a "stunning advance" in the citation for Wiles's Abel Prize award in 2016

The unsolved problem stimulated the development of algebraic number theory in the 19th and 20th centuries. It is among the most notable theorems in the history of mathematics and prior to its proof was in the *Guinness Book of World Records* as the "most difficult mathematical problem", in part because the theorem has the largest number of unsuccessful proofs.

Algorithm is a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer. For example, the *Python* code I used to check Collatz conjecture

```
num = 10
i=0
while num != 1:
   if (num % 2) == 0:
      num = num/2;
   else:
      num=num*3+1
   i=i+1
print (i)
```

is actually a set of rules to reduce any number to 1 using a few simple calculations. (Note, that in programming “=” is an assignment operator, not the equality sign.)
Here the algorithm to calculate *x* if *a* is known.



a	$1\frac{2}{7}$	$2\frac{3}{7}$	$2\frac{4}{7}$	$3\frac{1}{7}$	$4\frac{6}{7}$	$5\frac{5}{7}$	$10\frac{3}{7}$
x							

Algorithms are used to solve problems if an algorithm for solving a particular problem exists. If the problem has never been solved, there is no existing algorithm, and the goal becomes to create one. Write an algorithm to solve the problem:

How many liters of a 70% alcohol solution must be added to 50 liters of a 40% alcohol solution to produce a 50% alcohol sanitizing solution?

1. Find out how much alcohol is in 50 liters of 40% alcohol.

   ($50 : 100 \cdot 40 = 0.4 \cdot 50 = 20$
   There is 20 liters of alcohol is in 50 liters of solution.)
2. In x (added) liters of 70% solution there are $0.7x$ liters of alcohol and $0.3x$ of water.
3. In the final solution the volume of alcohol should be equal to a volume of water.

$$20 + 0.7x = 30 + 0.3x$$
$$0.7x - 0.3x = 30 - 20 = 10$$
$$0.4x = 10$$
$$x = 10 : 0.4 = 25$$

4. Answer: 24 liters of 70% alcohol solution should be added to 50 liters of 40% solutions to get 50% solution.
5. Check.
   Total volume is $50 + 25 = 75\ liters$
   Volume of alcohol is

$$20 + 0.7 \cdot 25 = 37.5$$

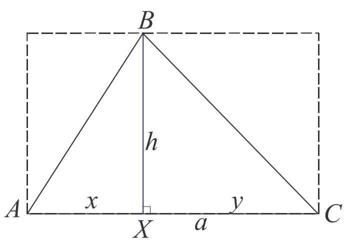
- The algorithm development allows to look at the problem in a rational way, which helps when it comes to solving different kinds of problems.
- The algorithm development helps to break down the problem into smaller pieces.
- Algorithms help reach the solution needed without going over the process repeatedly. You can follow the rules set up beforehand to reach the solution quicker.
- Algorithms use a definite procedure, and by using and optimizing them, people can solve problems much quicker.
- Algorithms are easy to debug because each step has its logical sequence.

Where do we use algorithms in our everyday life?

Cooking recipes, time management, small repair…

Area of a triangle.

The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

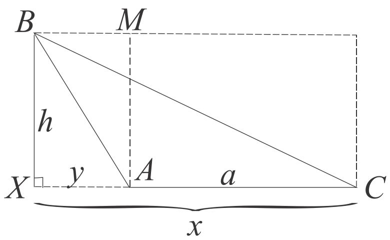


For the acute triangle it is easy to see.

$$S_{rec} = h \times a = x \times h + y \times h$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \qquad S_{\Delta XBC} = \frac{1}{2} h \times y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$



For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex.