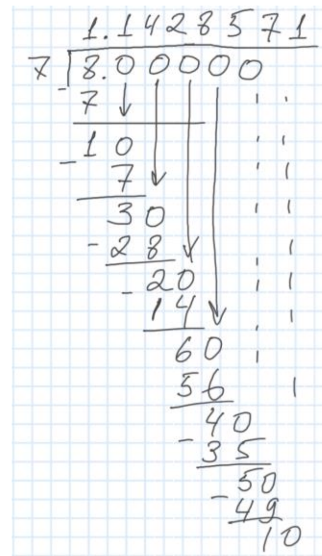
Classwork 15.

Rational numbers:

Numbers, that can be represented as a ratio of an integer to a natural number are rational numbers. Nonreducible fraction (if it's reducible, it can be reduces to a nonreducible form)

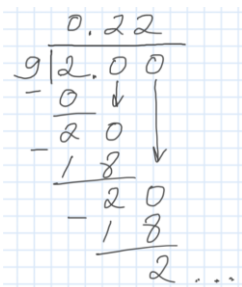
$$\frac{m}{n}; \qquad m \in Z,\ \ n \in N$$

Z is a set of integers $(0, \pm 1, \pm 2, \ldots)$, N is a set of natural numbers (1, 2, 3…) to avoid division by 0.



$$\frac{-2}{9} = -\frac{2}{9}$$

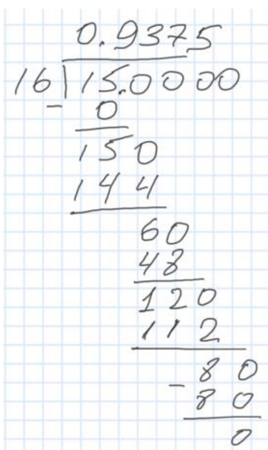
To represent a fraction (ratio) as decimal, we can just divide numerator by denominator:



$$\frac{2}{9} = 2:9 = 0.2222\ldots = 0.\overline{2}$$

$$\frac{8}{7} = 8:7 = 1.1428571\ldots = 1.\overline{142857}$$

(1428571) is a period of this decimal and will repeat infinitely many times.



$$\frac{15}{16} = 15:16 = 0.9375$$

At each step we either can get a remainder 0, then the division is complete. Or, the remainder repeats, and then we have to repeat steps again and again.

In the first case we got a finite decimal, in the latter one decimal is infinite periodic decimal. Finite decimal is a fraction with denominator where only 2 and/or 5 are prime factors.

$$\frac{15}{16} = \frac{15}{2^4} = \frac{15 \cdot 5^4}{2^4 \cdot 5^4} = \frac{9375}{10^4} = \frac{9375}{10000} = 0.9375$$

Decimals which are not infinite periodical (finite decimal can be cidered as decimal with 0 as periodical digit) can't be represented as ratio of integer and natural number. For example, 0.1010010001000010000001….can't be received by division of one whole number by another.

Such numbers are called irrational. Together with rational numbers they form the set of real numbers.

Transformation from decimal to fractional (rational) notation is easy for finite decimals based on a place value of digits:

$$0.3 = \frac{3}{10}; \quad 1.03 = 1 + 0.03 = 1 + \frac{3}{100} = 1\frac{3}{100} = \frac{103}{100};$$

$$0.123 = \frac{123}{1000} = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} = \frac{100}{1000} + \frac{20}{1000} + \frac{3}{1000}$$

For the infinite periodical decimals, the process is not so straightforward.

For $0.\overline{2}$:

Let's use *x* a variable and say

$x = 0.\overline{2}$. Then, we can multiply this *x* by 10. $10x = 0.\overline{2} \cdot 10 = 2.\overline{2}$

$$10x - x = 9x$$

$$10x - x = 2.\overline{2} - 0.\overline{2} = 2$$

$$9x = 2; \quad x = 2:9 = \frac{2}{9}$$

Another example from above:

$x = 1.\overline{142857}; \quad 1000000x = 1142857.\overline{142857}$

$1000000x - x = 1142857.\overline{142857} - 1.\overline{142857}$

$999999x = 1142856;$

$$x = \frac{1142856}{999999} = \frac{2^3 \cdot 3^3 \cdot 11 \cdot 13 \cdot 37}{3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37} = \frac{8}{7}$$

Rational numbers have the same properties of addition and multiplication as natural numbers.

Exercises:

1. Write decimal representation of fractions:

$$\frac{4}{9}; \quad \frac{17}{25}; \quad \frac{63}{75}; \quad \frac{5}{16}; \quad \frac{2}{11};$$

1. Represent the periodic infinite decimals as fractions:

$$0.\overline{8}, \qquad 0.\overline{4}, \qquad 0.\overline{18}, \qquad 0.\overline{125}, \qquad 0.1\overline{25}, \qquad 23.5\overline{13}$$

2. Multiply

$$(2x+3)(x+xy+2); \qquad (2-y)(y^2+y);$$

3. Solve equations:

$$2x + 3(4 - x) = 23 + 4x; \qquad 3x - 4 = 2(3x - 4)$$