Classwork 9.

$$\frac{(999^{-1}-1000^{-1})(999^{-1}+1000^{-1})}{(1000^{-1}-999^{-1})^2}=$$

Multiplication of two expressions.
To multiply an expression by a number or variable, we can use the distributive property:

$$a(a+2)=\underbrace{(a+2)+(a+2)+\cdots+(a+2)}_{a\ times}=\underbrace{a+a+a+\cdots+a}_{a\ times}+\underbrace{2+2+\cdots+2a}_{a\ times}$$

$$=a\cdot a+2a=a^2+2a$$

If we need to multiply two expressions

$$(a+2)\cdot(a+3)$$

We can use a substitution technic, we will substitute one of the expressions with a variable, for example, instead of $(a+2)$ we can use u.

$$(a+2)=u$$

And then we will multiply

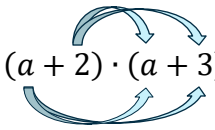
$$u\cdot(a+3)=u\cdot a+3u$$

We know, that actually u should not be there, $(a+2)$ should.

$$u\cdot(a+3)=(a+2)\cdot a+3(a+2)$$

We know how to multiply an expression by a variable (or number):

$$(a+2)\cdot a+3(a+2)=a\cdot a+2a+3a+3\cdot 2=a^2+5a+6$$



$$(a+2)\cdot(a+3)=a\cdot a+3a+2a+3\cdot 2=a^2+5a+6$$

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There are a few very useful products:

$$(a+b)^2=(a+b)\cdot(a+b)=a\cdot a+a\cdot b+b\cdot a+b\cdot b=a^2+2ab+b^2$$

What can be substituted for a an b?
Let's do a few examples:

$$(2+x)^2=(2+x)(2+x)=2\cdot 2+2\cdot x+x\cdot 2+x\cdot x=2^2+2x+2x+x^2$$
$$=x^2+2\cdot 2x+4=x^2+4x+4$$

$$(ab+2y)^2 = (ab+2y)(ab+2y) = ab\cdot ab + ab\cdot 2y + 2y\cdot ab + 2y\cdot 2y$$
$$= a^2b^2 + 2yab + 2yab + 4y^2 = a^2b^2 + 4yab + 4y^2$$

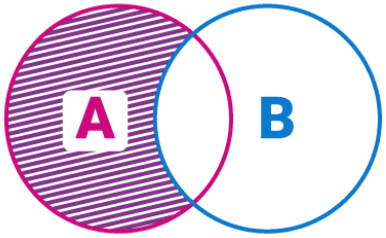
$$(a-b)(a+b) = a\cdot a + a\cdot b - a\cdot b + b\cdot b = a^2 - b^2$$

Revue of sets.

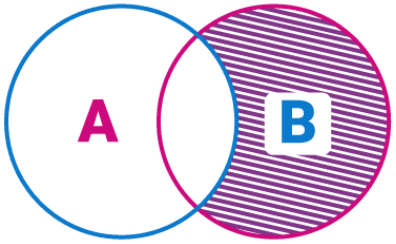
$\in$	element belongs to a set	$\not\subset$	one set is not a subset of another set
$\notin$	element does not belong to a set	$\cap$	intersection of two sets
$\subset$	one set is a subset of another set	$\cup$	union of two sets
$\varnothing$	empty set		

Let's introduce a few new concepts of the set theory.
The difference of two sets is A and B are all elements which belong to the set A, but don't belong to the set B:

$A - B$



$B - A$



1. Multiply.
   $a.$ $(a+2)(a+2);$
   $b.$ $(3+y)(y+4);$
   $c.$ $(3+x)(3-x);$
   $d.$ $(x-y)(x+y);$
   $e.$ $(2a+c)(a+ac);$
   $f.$ $(a+1)(a+3);$
   $g.$ $(c+d)(c-2d);$
   $h.$ $(y-2)(3-y);$
   $i.$ $(x-m)(x-m);$
   $j.$ $(2d+3l)(2d+3l);$

2. If A = { 3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, and C = {11, 13, 15}, then find B ∩ C and A ∩ B ∩ C.

3. In a group of 60 kids, 27 like orange juice and 42 like apple juice and each person likes at least one of the two drinks. How many like both orange and apple juice?

4. Let A and B be two finite sets such that set A has 20 elements and set B has 28 elements, $(A \cup B)$ has 36. How many elements does $(A \cap B)$ have?

5. If number of elements of the difference of sets A and B is 18, Unit of these two sets has 70 elements, and their intersection has 25 elements. How many elements does set B have?

6. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?