Classwork 6. Algebra.

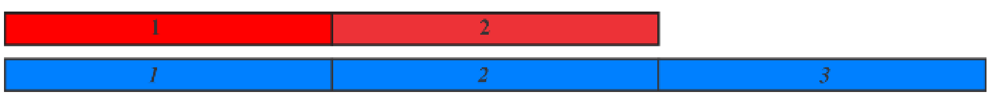
Algebra.

Ratio and proportions.

The ratio of two numbers indicates how many times one number is larger than another or which part of one number the other number is.

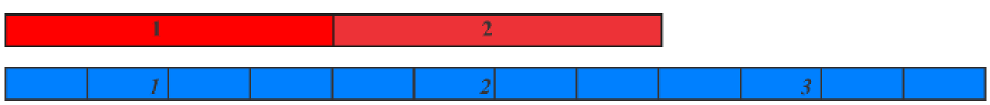
Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)

Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.



Step 3. Let's compare the number of sold and leftover balloons.

922

$1686 - 922 = 764$

Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$\frac{2}{3} \cdot 948 = 632$.Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations? Let's try.

$$G + B + R = 1686$$

$$3R = 2B$$

$$\frac{1}{2}G + R + \frac{1}{4}B = 922$$

$$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$$

$$R - \frac{1}{2}B = 158$$

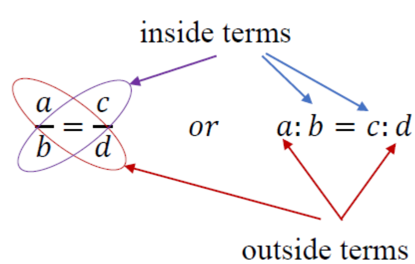
$$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$$

1 percent of quantity is a $\frac{1}{100}$ th part of it.

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3: 2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.
Proportions have several interesting features.



1. The products of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ac}=\frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b}=\frac{c}{a}$$

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ab}=\frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$

4. Two inside terms can be switched as well.

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c}=\frac{b}{d}$$

5. Also, several other new proportion can be created.

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a\pm b}{b}=\frac{c\pm d}{d}$$

(The sign $\pm$ is used to show that both, addition and subtraction, can be used)

Let's prove one of the statements:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b}+1=\frac{c}{d}+1 \quad \Leftrightarrow \quad \frac{a}{b}+\frac{b}{b}=\frac{c}{d}+\frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b}=\frac{c+d}{d}$$

6. Another proportion:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d}=\frac{c}{d}=\frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$

We know form (4) that

$$\frac{c}{a}=\frac{d}{b}$$

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}=\frac{d}{b}$$

Going back to the jam problem above. We got the simple equation

$$\frac{3}{2}=\frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x=27\cdot 2$$

$$x=\frac{27\cdot 2}{3}=\frac{3\cdot 9\cdot 2}{3}=18$$

1. To do her homework, Julia solved math problems, wrote an essay, and did a history project. It took her 2 hours and 15 minutes to finish all the assignments. The ratios of the times she spends doing math, writing the essay, and doing history project are 3:2:1. How much time did she spend for each of her subjects?

2. A book is 25% more expensive then a notebook. How many percent the notebook is less expensive than the book?

3. Solve the following equations (hint: use the property of proportions):

$$a.\ \frac{x}{7.2} = \frac{1\frac{1}{9}}{0.25}; \qquad b.\ \frac{2\frac{1}{3}}{0.6x} = \frac{2.5}{1\frac{2}{7}}; \qquad c.\ \frac{\frac{7}{12}}{0.14} = \frac{50x}{4.8}; \qquad d.\ \frac{1\frac{3}{17}}{13.75} = \frac{2\frac{2}{11}}{3x}$$

4. Dry apricots contain 22% of water. How much fresh apricot were used to produce a 200 g package of dry apricot if fresh apricots contain 85% of water?

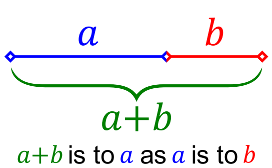
Famous ratios.

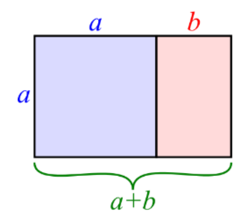
i. Let's measure the circumference and the diameter of a circle.

$$\frac{l}{d} = \pi$$

ii. Golden ratio:

$$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$$

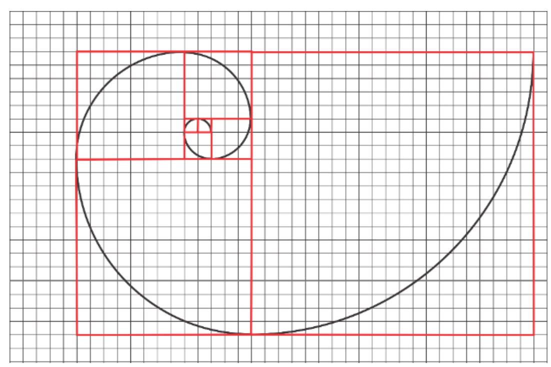


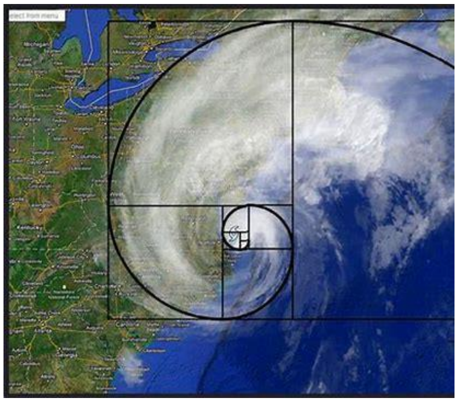


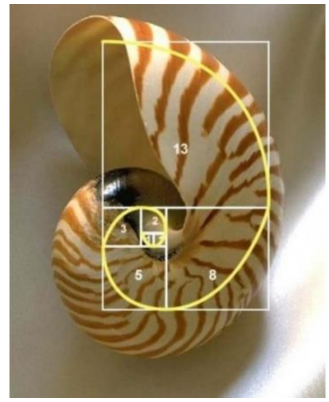
Fibonacci sequence:

$$1, 1, 2, 3, 5, 8 \ldots..$$

$$F_n = F_{n-1} + F_{n-2}$$







1. Simplify:

(a) $(2z^2 \cdot 3z^3 \cdot z)^2$

(b) $\left(\frac{5g^4b^5}{4g^2b^3}\right)^3$

(c) $2x^2 \cdot x^3 - x^7 \div x^2$

(d) $\frac{(-ab)^8}{(ab)^2}$

(e) $\frac{18^{n+3}}{3^{2n+5} \cdot 2^{n-2}}$

(f) $\left(\frac{3ab^3}{15b}\right)^2 \cdot \frac{75c}{a^2b^6}$

2. Let $x = a^3 \cdot b^2$, $y = \frac{b^5}{a^2c^4}$, and $z = \frac{c^3}{ab}$. Express in terms of a, b, c:

(a) xyz

(b) $x^2y^3z^4$

(c) $\frac{xy}{z}$

1. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$, what is the value of ab? of a/b?
2. In how many zeroes does the number $4^{15}5^{26}$ end?
3. Simplify:

(a) $(4c^2 \cdot c^3)^3$

(b) $\left(\frac{8dg^2}{3d^3g^4}\right)^3$

(c) $((x^2y)^3)^4$

(d) $\frac{26(a^2b)^4}{65a^3b^2c^3}$

(e) $\left(\frac{9a^7b^6}{45a^3b}\right)^4$

(f) $\left(\frac{3a^5b^2}{21ab}\right)^4 \cdot \frac{7^4}{a^{16}b^2}$

4. Let $x = a^3 \cdot b^2$, $y = \frac{b^5}{a^2c^4}$, and $z = \frac{c^3}{ab}$. Express in terms of a, b, c:

(a) $(xy)^2z$

(b) $\frac{x}{y}$

(c) $\frac{x^3y^2}{xy^2z^3}$

9. Write the following numbers in regular form:

(a) $9.21 \times 10^6 =$

(b) $1.527 \times 10^4 =$

(c) $5.3459 \times 10^3 =$

(d) $7.527 \times 10^2 =$

5. Suppose \$100 is deposited into an account and the amount doubles every 8 years. How much will be in the account after 40 years? Express your answer using powers.
6. At the beginning of an epidemic, 50 people are sick. If the number of sick people triples every other day, how many people will be sick at the end of 2 weeks? Express your answer using powers.
7. About how many hydrogen atoms are there in one gram of hydrogen?
8. Write the following numbers using scientific notation.
   (a) the distance from Earth to Pluto is ≈ 7,527,000,000 km;
   (b) the distance from Earth to the star Sirius is ≈ 81,900,000,000,000 km;
   (c) the distance from Earth to Vega is ≈ 249,500,000,000,000 km;
   (d) the distance from Earth to the Andromeda Nebula is ≈ 2,000,000,000,000,000,000,000 km.
   (e) the area of the Pacific Ocean is ≈ 178,684,000,000 km^2