Math 5b. Classwork 4.

Statements, which describe a property of a set of objects is categorical statement.

"Any (all) natural number is divisible by 3"

is a categorical statement (it's about a category "natural numbers") and it is a false statement. It is very easy to show: 5 is a natural number and it is not divisible by 3. It means that not any natural number is divisible by 3, some of the them are not disable by 3 and only one such example is enough to prove that the statement is false.
"The sum of any even numbers is an even number" is a categorical statement. The statement is about the category "sum of two arbitrary even numbers", and it describe a property "divisibility by 2". We can ether prove it wrong by showing at least one example of the odd sum of two even number, or prove it true by reasoning. It is not enough to show several examples to prove that this statement is true and of course there are no example to prove it wrong.

Prove. Any even number can be represented as $2k$ (or $2n$) where $k, n \epsilon N$

$$2k + 2n = 2(k + n), \quad k, n \epsilon N$$

So, the sum is divisible by 2, or even number.

Each week contains 7 days.
The result of multiplying any number by 0 is 0.
The sum of to numbers does not depend of the order of addends.
The perimeter of a rectangle equals the sum of the length of its sides.
The sum of two natural numbers is divisible by 3.
Product of two natural numbers is greater than its sum.

To negate the statement, we have to make the negation which is false. What do you think such statement will look like?

For example, the statement "all birds can fly" can be negated, as "not all bird can fly" or we can say "there are birds that can't fly" , there is at least one bird that can't fly,
Can you do the negation of the statement above?

Another important kind of statements are statements about existence of at least one object with certain property. For example:

There is a natural number k, *such that* $57 = 3k$
There is a natural number x, *such that* $(2x + 3) : 7 = 11$
There are people older than 100 years.
The product of two natural numbers can be grater than its sum.
The sum of two natural numbers is not always divisible by 3.

This kind of statements can be proven with an example.
Can you construct the negation of these statements?

Rational numbers.

Positive rational number is a number which can be represented as a ratio of two natural numbers:

$$a = \frac{p}{q}; \qquad p, q \in N$$

As we know such number is also called a fraction, p in this fraction is a nominator and q is a denominator. Any natural number can be represented as a fraction with denominator 1:

$$b = \frac{b}{1}; \;\; b \in N$$

Basic property of fraction: nominator and denominator of the fraction can be multiplied by any non-zero number n, resulting the same fraction:

$$a = \frac{p}{q} = \frac{p \cdot n}{q \cdot n}$$

In the case that numbers p and q do not have common prime factors, the fraction $\frac{p}{q}$ is irreducible fraction. If $p < q$, the fraction is called "proper fraction", if $p > q$, the fraction is called "improper fraction".
If the denominator of fraction is a power of 10, this fraction can be represented as a finite decimal, for example,

$\frac{37}{100} = \frac{37}{10^2} = 0.37,\qquad \frac{3}{10} = \frac{3}{10^1} = 0.3,\qquad \frac{12437}{1000} = \frac{12437}{10^3} = 12{,}437$

$$10^n = (2 \cdot 5)^n = 2^n \cdot 5^n$$

$$\frac{2}{5} = \frac{2}{5^1} = \frac{2 \cdot 2^1}{5^1 \cdot 2^1} = \frac{4}{10} = 0.4$$

$$\begin{array}{r|l} & 0.875 \\ 8 & 7.000 \\ & -6.4 \\ & \;\;\;60 \\ & \;\;-56 \\ & \;\;\;\;\;40 \\ & \;\;\;-40 \\ & \;\;\;\;\;\;\;0 \end{array}$$

Therefore, any fraction, which denominator is represented by $2^n \cdot 5^m$ can be written in a form of finite decimal. This fact can be verified with the help of the long division, for example $\frac{7}{8}$ is a proper fraction, using the long division this fraction can be written as a decimal $\frac{7}{8} = 0.875$. Indeed,

$$\frac{7}{8} = \frac{7}{2 \cdot 2 \cdot 2} = \frac{7 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{7 \cdot 5^3}{2^3 5^3} = \frac{7 \cdot 125}{(2 \cdot 5)^3} = \frac{875}{10^3} = \frac{875}{1000} = 0.875$$

Also, any finite decimal can be represented as a fraction with denominator 10^n.

In other words, if the finite decimal can be represented as an irreducible fraction, the denominator of this fraction will not have other factors besides 5^m and 2^n. Converse statement is also true: if the irreducible fraction has denominator which only contains 5^m and 2^n than the fraction can be written as a finite decimal. (Irreducible fraction can be represented as a finite decimal if and only if it has denominator containing only 5^m and 2^n as factors.)
If the denominator of the irreducible fraction has a factor different from 2 and 5, the fraction cannot be represented as a finite decimal. If we try to use the long division process, we will get an infinite periodic decimal.
At each step during this division we will have a remainder. At some point during the process we will see the remainder which occurred before. Process will start to repeat itself. On the example on the left, $\frac{5}{7}$, after 7, 1, 4, 2, 8, 5, remainder 7 appeared again, the fraction $\frac{5}{7}$ can be represented only as an infinite periodic decimal and should be written as $\frac{5}{7} = 0.\overline{714285}$. (Sometimes you cen find the periodic infinite decimal written as $0.\overline{714285} = 0.(714285)$).
How we can represent the periodic decimal as a fraction?
Let's take a look on a few examples: $0.\overline{8}$, $2.35\overline{7}$, $0.\overline{0108}$.

```
   0.71428571...
 7|5.000
 -0 0
   5 0
 - 4 9
     10
   -  7
      30
    - 28
       20
     - 14
        60
      - 56
         40
       - 35
          50
        - 49
           10
           .....
```

1. $0.\overline{8}$.
$x = 0.\overline{8}$
$10x = 8.\overline{8}$
$10x - x = 8.\overline{8} - 0.\overline{8} = 8$
$9x = 8$
$x = \frac{8}{9}$

2. $2.35\overline{7}$
$x = 2.35\overline{7}$
$100x = 235.\overline{7}$
$1000x = 2357.\overline{7}$
$1000x - 100x = 2357.\overline{7} - 235.\overline{7}$
$= 2122$
$x = \frac{2122}{900} = \frac{1061}{450}$

3. $0.\overline{0108}$
$x = 0.\overline{0108}$
$10000x = 108.\overline{0108}$
$10000x - x = 108$
$x = \frac{108}{9999} = \frac{12}{1111}$

Now consider $2.4\overline{0}$ and $2.3\overline{9}$

$x = 2.4\overline{0}$
$10x = 24.\overline{0}$
$100x = 240.\overline{0}$

$100x - 10x = 240 - 24$
$x = \frac{240 - 24}{90} = \frac{216}{90} = 2.4$

$x = 2.3\overline{9}$
$10x = 23.\overline{9}$
$100x = 239.\overline{9}$

$100x - 10x = 239 - 23$
$x = \frac{239 - 23}{90} = \frac{216}{90} = 2.4$

Exercises.

1. Evaluate the following using decimals:

$a.\ 0.36+\frac{1}{2};\qquad b.\ 5.8-\frac{3}{4};\qquad c.\ \frac{2}{5}:0.001;\qquad d.\ 7.2\cdot\frac{1}{1000}$

2. Evaluate the following using fractions:

$a.\ \frac{2}{3}+0.6;\qquad b.\ 1\frac{1}{6}-0.5;\qquad c.\ 0.3\cdot\frac{5}{9};\qquad d.\ \frac{8}{11}:0.4;$

3. Evaluate:

$$a.\frac{5\frac{1}{7}}{3\frac{3}{14}};\qquad b.\ \frac{1\frac{1}{3}\cdot 2\frac{3}{11}\cdot 3\frac{1}{2}}{\frac{1}{2}\cdot 4\frac{1}{6}\cdot 3\frac{9}{11}};\qquad s.\frac{1\frac{1}{2}\cdot 2\frac{2}{3}\cdot 0.36}{0.6\cdot 2\frac{1}{4}\cdot 1\frac{1}{3}};\qquad d.\ \frac{0.38\cdot 0.17\cdot 2\frac{2}{15}\cdot 2.7}{5.1\cdot 3\frac{4}{5}\cdot 0.064}$$

4. True or false the following statements?

1) $12+17=29$; 2) $12+17\neq 29$; 3) $12+17>29$; 4) $12+17<29$;
5) $12+17=28$; 6) $12+17\neq 28$; 7) $12+17>28$; 8) $12+17<28$;
9) $12+17\geqslant 29$; 10) $12+17\leqslant 29$; 11) $12+17\geqslant 28$; 12) $12+17\leqslant 28$.

5. True or false the following statements?

$a.\ 12x-35y=1\quad (x=3, y=1)$

$b.\ 14x-26y=0\quad (x=6, y=3)$

$c.\ 2x-y>27\quad (x=14, y=5)$

$d.\ x+2y<649\quad (x=8, y=2)$

$e.\ 5x-6y>28\quad (x=8, y=2)$

$f.\ 3x-y\leq 210\quad (x=8, y=2)$

6. False or true?
Solar system has 8 big planets.
Rome is the capital of Spain.
Nile and Amazon River are two biggest rivers in Africa.
A square is a rectangle.

7. Which of the following statements are categorical statements?
Some type of plants and animals are listed in the list of endangered species.
All planets of the Solar system are rotating around the Sun in the same direction.

Some butterflies are yellow.
There are 22 books on the shelf.
Any natural number is greater than 0.

8. Find one counter example to prove the following statements wrong:
All natural numbers are greater than 1.
Any number divisible by 5 ends with digit 5.
All rivers of the United States flow into the Pacific Ocean.
All marine animals are fish.
All American cities lie south of 50° latitude line.

9. The following statements are proven. Can you tell which prove is right and which is wrong?
All natural numbers are divisible by 7, for example: 14:7=2.
Some proper fractions have denominator equals to 8, for example: denominator of the fraction $\frac{3}{8}$ is 8.
There are even numbers multiple of 3, for example 36 is multiple of 3, 36:3=12
Some nouns in English language contain 5 letters, for example “table”.
All verbs in English language start with letter “w”, for example “ (to) write”.
There are books written by Joanne Rowling, for example Harry Potter and the Philosopher's Stone.

10. You meet two inhabitants: Ned and Zoey. Zed says that it’s false that Zoey is a knave. Zoey claims, ‘I and Ned are different.’ Can you determine who is a knight and who is a knave?

11. You meet two inhabitants: Sue and Marge. Sue says that Marge is a knave. Marge claims, ‘Sue and I are not the same.’
Can you determine who is a knight and who is a knave?

12. Represent the following fractions as decimals:

$a.\ \frac{3}{2000},$ $\quad d.\ \frac{7}{4};$ $\quad g.\ \frac{123}{20};$

$b.\ \frac{17}{40};$ $\quad e.\ \frac{3}{2};$ $\quad h.\ \frac{783}{540};$

$c.\ \frac{28}{140};$ $\quad f.\ \frac{9}{5};$ $\quad i.\ \frac{324}{25};$

13. Write as a fraction

$a.\ 0.\overline{3},\quad b.\ 0.3,\quad c.\ 0.\overline{7},\quad d.\ 0.7,\quad e.\ 0.1\overline{2},\quad f.\ 0.\overline{12},\quad g.\ 0.$