

Math 5b, homework 3.



Exercises:

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave. . .

Try to solve the following problems about knights and knaves:

1. You meet two inhabitants: Peggy and Zippy. Peggy tells you that ‘of Zippy and I, exactly one is a knight’. Zippy tells you that only a knave would say that Peggy is a knave.
Can you determine who is a knight and who is a knave?
2. You meet two inhabitants: Marge and Zoey. Marge says, ‘Zoey and I are both knights or both knaves.’ Zoey claims, ‘Marge and I are the same.’
Can you determine who is a knight and who is a knave?
3. You meet two inhabitants: Ted and Zeke. Ted claims, ‘Zeke could say that I am a knave.’ Zeke claims that it’s not the case that Ted is a knave.
Can you determine who is a knight and who is a knave?
4. Find a two-digit number that is 5 times the sum of its digits.
5. Fill up the empty places for the equality to hold (use the distributive property):
 - a. $(35 + a) \cdot 2 = \square + 2a$
 - b. $\square \cdot (11 - 7) = \square - 21b$
 - c. $9c + \square = (9 + 1) \cdot c$
 - d. $5 \cdot (a + 7) = 5 \cdot \square + \square \cdot 7$
 - e. $10 \cdot (\square - \square) = 140 - 10x$
 - f. $(\square - \square) \cdot 20 = 40x - 60$

Example:

$$(\square - \square) \cdot 20 = 40x - 60; \quad (2x - 3) \cdot 20 = 40x - 60$$

6. Evaluate:

a. $4\frac{1}{6} \cdot \left(1\frac{1}{2} - \frac{3}{5}\right) + \left(\frac{3}{4} + \frac{5}{6}\right) \cdot 6;$

b. $\left(6 - 2\frac{4}{5}\right) \cdot 3\frac{1}{8} - 1\frac{3}{5} : \frac{1}{4};$

c. $24 - \left(3\frac{3}{5} - 1\frac{7}{9}\right) : \left(\frac{1}{2} - \frac{1}{3}\right);$

d. $4 \cdot \left(2\frac{1}{2} + 1\frac{3}{4}\right) - \left(6\frac{2}{3} + 4\frac{4}{5}\right) : 2;$

7. A bar of soap weighs as much as $\frac{3}{4}$ of an identical bar plus $\frac{3}{4}$ of a pound. How much does the bar of soap weigh?

8. A truck can cover distance between two cities in 10 hours. A fast car, which goes 10 miles per hour faster than the truck, can cover the same distance in 8 hours. What is the distance? [Hint: if the speed of the truck is x mph, then the distance is equal to $10x$ miles. On the other hand....]