

MATH 5: HOMEWORK 26: GAMES.

1. Recall the game of Nim: A number of beans are lined up. Two players take turns picking up the beans. In one turn, a player can pick up 1, 2, 3, 4, or 5 stones (no more than 5, no less than one). The player to pick up the last beans loses the game. If there are 12 beans initially, who wins and how? If there are 15, who wins and how? 100 beans? 1000 beans?
2. There is a row 100 chairs with two children initially sitting in the two chairs at the ends of the row. In turns, each child moves over towards the other by either one or three places. Thus continues until one cannot make a move any more (e.g. the two children are sitting in neighboring chairs). The child The that cannot move loses. Which of the two children (the one who starts or the one who moves second) has a winning strategy? What happens if there are 101 chairs? (Hint: first think what happens if there are 4 or 5 chairs. Try to find and invariant).
3. Two children break, alternately, an 6×8 piece of chocolate into smaller pieces (leaving any 1×1 piece intact). The child that can not make a break loses. Which child is losing?
4. A rook is placed on bottom-left square of a 8×8 chessboard. Two players alternate moving the rook any number of squares, but only to the right or upwards. The player who cannot move loses. Which player can win and how? (Hint: think about the diagonal)