MATH 5: HANDOUT 20.5
MATH BATTLE: REVIEW

1. There are five heads and fourteen legs in a family. How many people and how many dogs are in the family?

2. Is 12345 divisible by 3? By 9?

3. Your ceiling fan is going, somebody pulls the cord once, and you want to return it to the original setting, but you don't know if it is a 2-setting or a 3-setting fan. What is the smallest number of times you need to pull to be sure the fan is back in its original setting? Is there a way to do this if the fan might be 4-setting? Any number?

4. You know how two people can divide a pie, fairly: One cuts the pie into two pieces, and the other gets to choose which piece he wants. How could 3 people divide a pie fairly (also guarding against conspiracies)? Is there a way for more than 3 persons?

5. A boat travels with a speed of 15 mph in still water. In a river flowing at 5 mph, the boat travels some distance downstream and then returns. What is the ratio of average speed to the speed in still water?

6. A train leaves New York for Washington every hour on the hour. A train leaves Washington to New York every hour on the half-hour. The trip takes five hours each way. As you ride from NY to Washington, how many trains bound from Washing to NY would you pass?

7. Solve the following puzzle (different letters stand for different digits):

$$\begin{array}{r} \text{SEND} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array}$$

8. Suppose that you visit the Island of Knights (always tells the truth) and Knaves (always lies) because you have heard a rumor that there is gold buried there. You meet a native and you wish to find out from him whether there really is gold there, but you don't know whether he is a knight or a knave. You are allowed to ask him only one question answerable by 'yes' or 'no'. What would you ask?