MATH 5: HANDOUT 18
CHOOSINGS AND PERMUTATIONS.

Choosing with repetitions

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet?

Answer: there are 26 possibilities for the first letter, 26 for the second, and so on — so according to the product rule, there are $(26)^3$ possible combinations.

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are allowed: same item can be used more than once (e.g., same letter may appear several times in a combination).

Then there are n^k ways to do it.

Choosing without repetitions

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet if no letter can be used more than once?

Answer: there are 26 possibilities for the first letter; after we have chosen the first letter, it leaves only 25 possibilities for the second letter; after choosing the second, we only have 24 possibilities left for the third. So the answer is $26 \times 25 \times 24$

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are not allowed: no item can be used more than once.

Then there are $n(n-1)\dots(n-k+1)$ ways of doing it (the product has k factors). This number is usually denoted

$$_kP_n = n(n-1)\dots(n-k+1)$$

Factorials and permutations

Recall from last time: if we are choosing k objects from a collection of n so that a)order matters and b)no repetitions allowed, then there are

$$_kP_n = n(n-1)\dots \quad (k \text{ factors})$$

ways to do it.

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to put n objects in some order:

$$n! = {_nP_n} = n(n-1)\cdot\dots\cdot 2\cdot 1$$

(reads n factorial).

For example: there are $52!$ ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2\cdot 3\cdot 4 = 24$, $5! = 120$, $6! = 620$

Using factorials, we can give a simpler formula for $_kP_n$:

$$_kP_n = \frac{n!}{(n-k)!}$$

For example:

$$_4P_6 = 6\cdot 5\cdot 4\cdot 3 = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = \frac{6!}{2!}$$