

MATH 5: HANDOUT 17
BEGINNING PROBABILITY – 2.

PRODUCT RULE

Question: we roll two dice. What is the probability of rolling a 5 and a 6?

Answer: There are two ways of getting a 5 and a 6: as pair (5, 6) (5 on die number 1, 6 on die number 2) or as (6, 5) (6 on die number 1, 5 on die number 2). Thus, the answer is $\frac{2}{36}$.

Question: we roll two dice. What is the probability of getting sum of two numbers equal to 4?

Answer: there 3 ways of getting sum 4: (1, 3), (2, 2), (3, 1). Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. $(\frac{1}{2})^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or 1/10 of 1%).

Question. If toss a coin 10 times, what is the probability that all will be tails?

Answer. The same.

Question. If we toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer. Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

PERCENTAGES AND FRACTIONS

So far we have mostly expressed probabilities as fractions. They can also be written as decimal numbers (between 0 and 1): for example, $\frac{1}{5} = \frac{2}{10} = 0.2$. It is also common to express probabilities as percentages: by definition,

$$1\% = \frac{1}{100} = 0.01$$

so $x\% = \frac{x}{100}$. For example, $3\% = \frac{3}{100} = 0.03$, and $1.5\% = \frac{1.5}{100} = \frac{15}{1000} = 0.015$.

This conversion is necessary when you multiply probabilities as the following example shows:

Question. The probability of winning in a certain game is $p = 5\%$. What is the probability of winning two times in a row?

Answer. According to multiplication rule it is $p \times p = p^2$. However, the answer $5\% \times 5\% = 25\%$ is wrong. Correct answer is $\frac{5}{100} \times \frac{5}{100} = \frac{25}{10,000} = 0.0025$.

To convert from decimals to percent, multiply by 100:

$$p = (p \times 100)\%$$

For example, $\frac{1}{5} = 0.2 = (0.2 \times 100)\% = 20\%$