MATH 5: HANDOUT 15
DIFFERENCE OF SQUARES. SQUARE ROOT. PYTHAGOREAN THEOREM.

Difference of Squares

There is an important formula that allows you to factor a difference of squares:

$$a^2 - b^2 = (a-b)(a+b)$$

Square roots

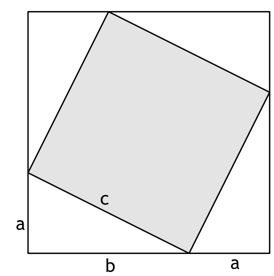
Square root of a is a number whose square is equal to a. For example: square root of 25 is 5, because $5^2 = 25$.

- Notation: square root of number a is commonly denoted $\sqrt{a}$.
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b}$ is **not** equal to $\sqrt{a} + \sqrt{b}$.

Pythagorean theorem: In a right triangle with legs a, b and hypotenuse c, one has

$$a^2 + b^2 = c^2 \qquad \text{ot} \qquad c = \sqrt{a^2+b^2}.$$

Proof: Consider the following picture:



In this square, the total area is

$$(a+b)\times(a+b) = a\times(a+b) + b\times(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

On the other hand, the area of each triangle is $\frac{1}{2}ab$, and the area of shaded square is c^2. Thus, we get

$$a^2 + 2ab + b^2 = 4\times\tfrac{1}{2}ab + c^2$$

which gives $a^2 + b^2 = c^2$. □

For example, in a square with side 1, the diagonal has length $\sqrt{2}$.

It is possible — but not easy — to find a right triangle where all sides are whole numbers. The easiest such triangle is the triangle with sides $3, 4, 5$.

Power $\frac{1}{2}$

We know how to raise numbers into whole powers:

$$a^n = a\times\cdots\times a.$$

But what is $a^{\frac{1}{2}}$?

Example: Let's try to figure out what $4^{\frac{1}{2}}$ is:

$$4^{\frac{1}{2}}\times 4^{\frac{1}{2}} = 4^{\frac{1}{2}+\frac{1}{2}} = 4^1 = 4.$$

We can see that $4^{\frac{1}{2}}$ must be a number, such that if we multiply it by itself, we get 4. But this is just a square root of 4! So, we get:

$$4^{\frac{1}{2}} = \sqrt{4}.$$

In general, this is also true:

$$a^{\frac{1}{2}} = \sqrt{a}.$$