

MATH 5: WORKSHEET 9
POWERS OF 2. BINARY NUMBERS.

1. Solve the following equations:

(a) $5(x - 1) - 4 = 3x + 1$ (b) $\frac{2}{3}(x - 2) = -18$ (c) $|2x - 1| = 7$

2. When Dennis was 27, his son was three years old. Now his son's age is one third of Dennis' age. How is each of them now?

3. Find the sum $1 + 2 + 4 + \dots + 2^n$ (the answer, of course, will depend on n). [Hint: first try computing it for several small values of n : find $1 + 2$, then $1 + 2 + 4$, then $1 + 2 + 4 + 8$. See if you can notice a pattern. After this, try formulating a general rule.]

4. There are 15 samples of water from various wells. It is known that exactly one of them contains a dangerous chemical. A lab can test water for the chemical, but the analysis is time-consuming and expensive. Can you find the sample containing the chemical using fewer than 15 tests? [Hint: you can take a drop of water from each of several samples and send the mix for analysis; then you would know if the chemical was in one of these samples.]

5. Convert the decimal numbers to binary:

9, 12, 24, 38, 45

6. Convert the following binary numbers to decimal:

101, 1001, 10110, 11011, 10101

7. What is the largest number that can be written as a 5-digit binary number? (Hint: what is the smallest 6-digit binary number?)

8. In order to allow computers to deal with different languages, computer scientists have developed so-called Unicode, a standard list of symbols covering virtually all human languages, from Armenian to Vietnamese. In particular, it includes Latin letters, Cyrillic letters, Chinese characters (hanzi), Emoji, and more.

The latest revision of Unicode contains about 96,000 symbols. If we want to represent each of them by a sequence of 0 and 1, would it be enough to use 16 bits (0s and 1s) for each symbol? If not, what is the smallest number of bits per symbol one would need?