MATH 5: HANDOUT 8
NEGATIVE NUMBERS. ABSOLUTE VALUE

PUZZLES (CLASSICAL)

1. There is a round lake in South America. Every year, on June 1st, a Victoria Regia flower appears at its centre (its stem rises from the bottom,and its petals lie on the water like those of a water lily). Every day the area of the flower doubles, and on July 1st, it finally covers the entire lake, drops its petals, and its seeds sink to the bottom. On which date is the flower's area half the area of the lake?

2. A ranger walked from his tent 10km southwards, turned east, walked straight eastwards 10km more, met his bear friend, turned north and after another 10km found himself by his tent. What color was the bear and where did all this happen?

NEGATIVE NUMBERS: A REVIEW

Numbers to the left of zero on the number line are called negative. They are less than 0, and we write the "–" in front of them. The numbers to the right from zero are positive.

Addition and subtraction. If we add a positive number to any number, we move to the right along the number line. If we add a negative number to any number, we move to the the left along the number line. So, adding (-5) is moving 5 units to the left on the number line — which is the same as subtracting 5. This rule holds in general:

$$a + (-b) = a - b$$

Addition is commutative: $a + b = b + a$, whether numbers a, b are positive or negative. So $(-3) + 7 = 7 + (-3) = 7 - 3 = 4$.

Opposites. Pairs of numbers -1 and 1, -2 and 2, -3 and 3 etc. are called the opposites. They lie at the same distance from zero on the number line, but in the opposite directions. For any number x (whether positive or negative), we will denote by $-x$ the opposite of x. For example, $-(-2)$ is the opposite of negative 2, which is equal to 2.

ABSOLUTE VALUE

The distance of a number from zero on the number line is called the absolute value of a number. The symbol for absolute value is $|\ |$. For example, $|4| = |-4| = 4$. It holds in general: for any positive x, we have $|x| = |-x| = x$.

Equations with absolute values. An equation like $|x| = 5$ has two solutions: $x = 5$ and $x = -5$. An equation like $|x - 1| = 4$ also has two solutions: $x - 1 = 4$ (which gives $x = 5$) or $x - 1 = -4$, which gives $x = -4 + 1 = -3$.

1. Solve and graph the answer:$|x–1| > 2$.

Solution: $x > 3$ or $x < -1$. The graph would be the numberline with everything to the left of -1 and everything to the right of 3, not including the endpoints.

2. Solve and graph the answer: $3|x|–2 \leq 1$.

Solution: This is equivalent to $|x| \leq 1$. Thus $x \leq 1$ and $x \geq -1$. On the number line we would fill in -1 to 1, including the endpoints.

3. A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs .3 ounces more or less than the desired weight, the bag is rejected. Write this equation to find the heaviest and lightest bag the machine will approve.

Solution: $|x - 16| = 0.3$ where x is the bag's weight. Thus, $x = 15.7$ oz, or 16.3 oz.

4. Amtrak's annual passenger revenue for the years 1980–2000 is modeled approximately by the formula $R = -40|x - 11| + 990$ where R is the annual revenue in millions of dollars and x is the number of years since January 1, 1980. In what years was the passenger revenue $790 million?

Solution: $790 = -40|x - 11| + 990$ Solve for $|x - 11| - 200 = -40|x - 11|5 = |x - 11|$ So $x - 11$ is either equal to 5 or to -5, the two numbers with an absolute value of 5. Thus, $x = 6$ or 16 so in 1986 and 1996.

5. John was seven dollars short to buy a first math book, and Amanda lacked one dollars. They combined their money to buy one book to share, but even then they did not have enough. How much did the book cost?

Solution: Let the cost of the book be x dollars. Let John have j dollars and Amanda have a dollars.

$$j = x - 7 \qquad a = x - 1.$$

We know that $x > 7$ since John lacks 7 to buy it. We know also that $a + j = 2x - 8 < x$. Thus the book costs between 7 and 8 dollars.