

MATH 5: HANDOUT 4
ALGEBRAIC EXPRESSIONS AND WORD PROBLEMS

BRAIN TEASERS

These puzzles are by Raymond Smullyan and are from The Tonight Show with Johnny Carson in 1982.

1. John, James and William are triplets who look the same. One day you meet one of them on the street, and you want to figure out if the person is John.

You know that John and James always lie, and William always tells the truth. You can ask a single question, whose answer has to be yes or no, and the question can have at most 3 words. What could you ask?

2. There are one hundred politicians at a party. Each politician is either honest or crooked, and they are not all of the same type—at least one is honest and at least one is crooked.

If you pick any two politicians at random, at least one in the pair is crooked. How many honest politicians are there?

TODAY'S MATERIAL

Today we discussed more rules for algebraic operations, involving subtraction:

$$a - (b + c) = a - b - c$$

$$a - (b - c) = a - b + c$$

$$a(b - c) = ab - ac$$

We also talked more about solving word problems using equations. Here are two examples of problems solved using equations:

Problem: An apple cost 9 cents, and an orange 15 cents. Elena bought some apples and oranges, 20 fruit in all, and paid \$2.64. How many apples and how many oranges did she buy?

Solution: Let a =number of apples; then number of oranges is $20 - a$. Thus, the total cost is $9a + 15(20 - a)$ cents. So we have an equation

$$9a + 15(20 - a) = 264$$

$$9a + 15 \times 20 - 15a = 264$$

$$300 + 9a - 15a = 264$$

$$300 - 6a = 264$$

$$300 - 264 = 6a$$

$$36 = 6a$$

$$a = 6$$

Problem: A messenger is sent from one city to another; he can travel 40 km a day. Next day, another messenger is sent, who can travel 45 km a day. When will he overtake the first messenger?

Solution: after x days, the first messenger has traveled $40x$ km, and the second one has travelled $45(x - 1)$ km (he started one day later!). Thus, we have an equation

$$40x = 45(x - 1)$$

$$40x = 45x - 45$$

$$0 = 45x - 40x - 45$$

$$45 = 45x - 40x = 5x$$

$$x = 9$$