

## MATH 5: HANDOUT 3 ALGEBRAIC EXPRESSIONS

In mathematics and other sciences we often use letters instead of numbers. Usually it is done to show that certain relationship will work for all numbers. Letters are also commonly used for unknown values. These letters are called *variables*.

Expressions involving both numbers and variables are called *algebraic expressions*.

Examples:  $3a$ ;  $7b + 8$ ;  $357 + 10x$ ;  $(65z - 459) \div 4$

In algebraic expressions we omit the sign of multiplication between a number and a variable. Instead of  $7 \times b$  we write  $7b$ , instead of  $10 \times z$  we write  $10z$ . In products, a number goes first, and then goes a variable. We do not write  $k \times 10$ , we write  $10k$ .

Using variables, we can write the basic rules for addition and multiplication as follows:

$$a + b = b + a \quad \text{commutative law for addition}$$

$$a + (b + c) = (a + b) + c \quad \text{associative law for addition}$$

$$ab = ba \quad \text{commutative law for multiplication}$$

$$a(bc) = (ab)c \quad \text{associative law for multiplication}$$

$$a(b + c) = ab + ac \quad \text{distributive law}$$

These laws can be used for simplifying calculations and rewriting expressions in a simpler form. For example:

$$\begin{aligned} 2x + 3 + 5 \times (x + 1) &= 2x + 3 + 5x + 5 && \text{“opening the parentheses”} \\ &= 2x + 5x + 3 + 5 = (2 + 5)x + 8 = 7x + 8 \end{aligned}$$

The operation we did in the last line — combining terms  $2x$  and  $5x$  into a single term  $7x$  — is very commonly used; it is called “collecting the like terms”. Note, however, that it is only possible if the terms contain the same variable: we can not collect like terms in an expression like  $2x + 7y$ .

We also discussed the method for solving simple equations. The main idea is that we start with a given equation and then transform it, making it simpler and simpler, until at the end we can find the value of the variable. In particular:

- Given an equation, we can or subtract add to both sides the same number. For example, we can replace equation  $3x + 5 = 20$  by  $3x = 15$  (obtained by subtracting 5 from both sides of the original equation)
- We can multiply or divide both sides of an equation by the same number. For example, we can replace  $3x = 15$  by  $x = 5$  (obtained by dividing both sides by 3).

**Homework: finish the worksheet from class**