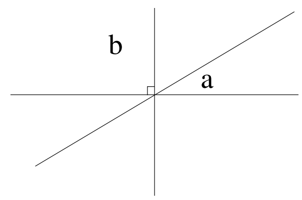
MATH 5: HANDOUT 02
REVIEW II

Review Topics

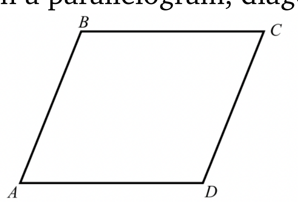
- Basic geometric concepts. Parallel lines. Angles.
- Sum of angles of a triangle and a polygon.
- Quadrilaterals: parallelogram, rectangle, square, rhombus.
- Areas. Area of triangle, trapezoid, parallelogram.
- Principle of the shortest way (angle of incidence = angle of reflection)

Problems

1. In the figure on the right, $\angle a = 30°$ and $\angle b$ is the right angle. Can you find the sizes of all other angles in the figure?

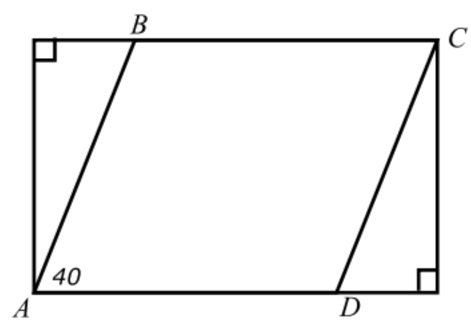


2. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C, \angle B = \angle D$.

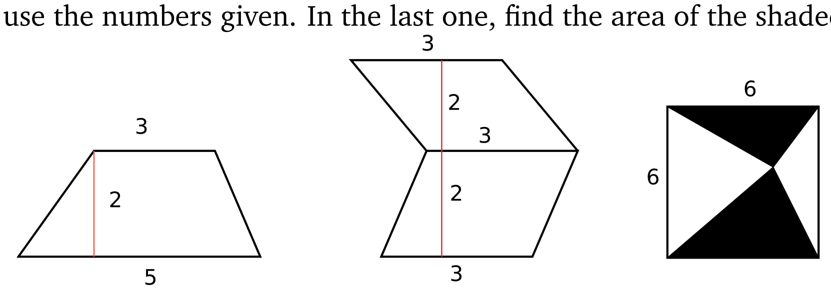


Show that it also works in the other direction: if in a quadrilateral, diagonally opposite angles are equal: $\angle A = \angle C, \angle B = \angle D$, then the quadrilateral must be a parallelogram.

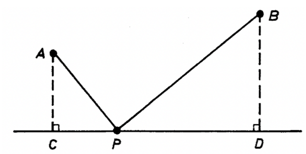
3. In the figure below, ABCD is a parallelogram, and $\angle DAB = 40°$. Can you find all other angles in the figure?



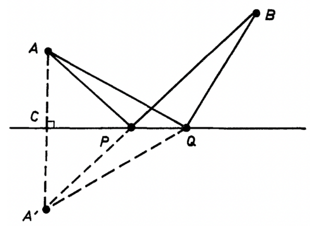
4. Compute the area of the figures below. The picture is not to scale, so do not try measuring the lengths - use the numbers given. In the last one, find the area of the shaded part.



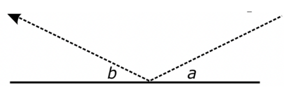
5. Bouboulina is in a race. She starts at point A, must touch the wall (pictured line) and must run to point B. At what point P should she touch the wall so that she has to run the shortest distance.



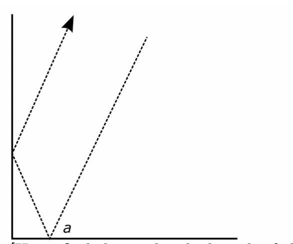
The solution is simple. Pretend the wall is a mirror. If you stand at B looking into the mirror will not see A but an image of A, A'. Where this image point is located is suggested by where the image is when looking directly into a mirror; it is symmetrically located with respect to the mirror. The shortest path is just a straight line between A' and B. Note, by the triangle inequality, any other path would be longer since the sum of any two sides of a triangle are greater than the third.



Note that the angle of incidence equals the angle of reflection. This is called Snell's law. e.g. for a beam of light or a billiard ball (the line being a mirror or the side of a pool table), the angle a is equal to the angle b.



6. How to build a reflective sign?



Show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one. [Hint: find the angle which each of these lines form with the horizontal (both make an angle of $90 - a$).]

Homework problems on back

1. Homework Problems

1. Is it true that any rectangle is also a parallelogram? Is it true that any parallelogram is a rectangle? Try to argue as carefully as you can.

2. To check whether a piece of paper is a square, John folds it along a diagonal. If the corners match, he decides it is a square. Is he right? What if he folds along both diagonals?

3. Cut two identical paper triangles (the easiest way to do it is to fold a sheet of paper in two and then cut). Can you put these two triangles together so that they form a parallelogram? Will your method always work? Why?

4. A boat has speed of 8 miles per hour (mph).
   (a) Two towns, A and B, are on the shores of a lake. How long would it take the boat to go from A to B and back if the distance between the towns is 10 miles?
   (b) Two other towns, C and D, also 10 miles apart, are on a river: C is upstream, D is downstream. The river flows at the speed of 2 mph. How long will it take the boat to go from C to D? from D to C?

5. A boat travels with a speed of 15 mph in still water. In a river flowing at 5 mph, the boat travels some distance downstream and then returns. What is the ratio of average speed to the speed in still water?

2. Extra Problems (for fun)

1. Consider any four points on the plane such that no three of them are collinear and so that they do not form a rectangle. Any subset consisting of three points form a triangle. (a) How many triangles can you make out of four points? (b) What is the minimum and maximum possible number of obtuse triangles (having an angle $> 90°$) defined by any such four points? Draw examples to exhibit this.

2. Find the angle between the two clock hands at 12:20.

3. Cut a triangle into 4 triangles, **any two** of which have a common boundary (not just a point, but a whole segment!).