Math 4b. Classwork 4.

Prime factorization

A number can be represented as a product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 2 \cdot 3 \cdot 6$$

Prime number is a number that has only two factors: the number itself and one.

Composite numbers are those numbers that have more than two factors.

Examples: numbers 2, 3, 5, 11, 19 are primes; number 42 is composite (because 42 = 6 × 7, so it has divisors/factors 6, 7.) Any number can be written as a product of several primes: if it is not prime, it can be written as a product of two smaller numbers, then we can repeat the same with these numbers. E.g.: 42 = 6 × 7 = 2 × 3 × 7.

Prime factorization is a decomposition of a natural number into the product of prime numbers.

Let's have a look at the example.
Number 360 has many factors. Let's take a pair which product makes 360:

$$360 = 36 \cdot 10$$

Each number on the right side can be written as a product of two factors:

$$360 = 6 \cdot 6 \cdot 2 \cdot 5$$

We can continue this process until all factors on the right side are prime.

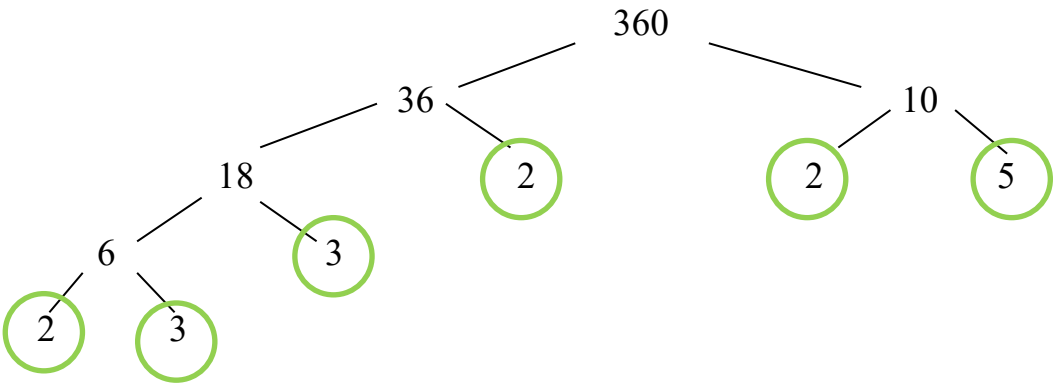
$$360 = 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 5$$

It is common to write a prime factorization in an ascending order: from smallest prime number to the largest like this:

$$360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

Any natural number has single unique prime factorization.

Prime factorization process can be done using **factor tree**:

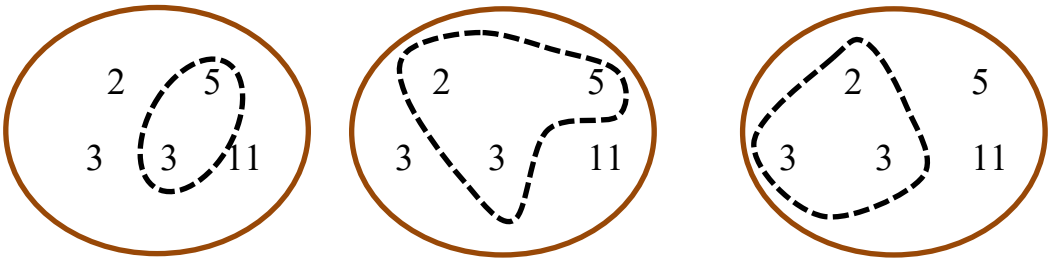


By doing prime factorization of a number one can learn a lot about this number's divisibility. Let's have a look at the example. The prime factorization of 990 is:

$$990 = 99 \cdot 10 = 9 \cdot 11 \cdot 2 \cdot 5 = 3 \cdot 3 \cdot 11 \cdot 2 \cdot 5$$

Knowing all prime factors of a number allows us to get a list of all other divisors/factors by choosing several prime factors and multiplying them.

990 990 990



15 is a factor 30 is a factor 18 is a factor

Can you name other factors of 990?

Homework Problems

1. Find all prime numbers of the following numbers (use prime factorization tree)
   66, 28, 128, 555, 1233

2. Represent numbers 64, 75, and 96 as a product of prime numbers.

3. Find the prime factorization of the number:

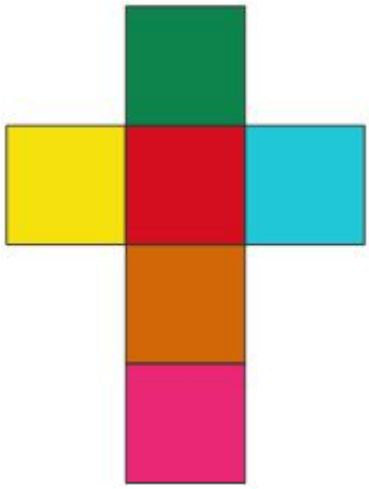
$$2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$$

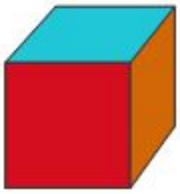
4. There are 4 children in the family. They are 5, 8, 13, and 15 years old and their names are Julia, Peter, Mary and Ellen. What is the age of each of them if one of the girls goes to kindergarten, Julia is older than Peter, and sum of ages of Julia and Mary is divisible by 3?

5. Rebecca wants to decorate the box with a birthday present for her friend Alice with a ribbon as shown in the picture. How long should the ribbon be if 90 cm should be left for the ends and the bow?

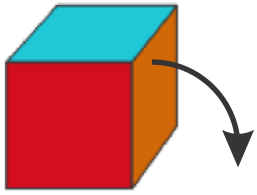
6. Prime factorization of a number is $2 \cdot 3 \cdot 5 \cdot 7$. Is this number divisible by:
   a) 6 b) 15 c) 4

7. On a picture on the right there is a surface of a cube.
   a) What do you think about the color of bottom side of this cube?

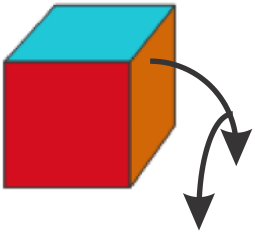




b) If you turn this cube ones following the arrow, what color of the upper side will be?



c) If you turn this cube one more time following the second arrow, what color of the upper side will be?



8*. In a remote village many years ago villagers successfully bred dragons. In a flock of 67 dragons one dragon breeder counted 48 Fire-Breathing Dragons, and another dragon breeder counted 47 Steam-Breathing dragons. Both swore there were no mistakes. How could it be? [Hint: graphic explanation using Venn diagram will be a good choice.]