

**MATH 10**  
**THE MATH BATTLE!**  
MARCH 12, 2023

1. Let us call a number between 0 and 999999 (inclusive) “lucky” if the sum of the first 3 digits is equal to the sum of the last 3 digits. Is the total number of lucky numbers even or odd?  
(we write all numbers as 6-digit ones, adding zeros in the beginning if necessary).
2. Let  $n!!$  be the product
$$n!! = n(n-2)(n-4)\dots$$
(for example,  $10!! = 10 \times 8 \times 6 \times 4 \times 2$ , and  $7!! = 7 \times 5 \times 3 \times 1$ ).  
Prove that  $2021!! + 2022!!$  is a multiple of 2023.
3. Andrew wrote on blackboard two numbers: 20 and 100. After that, he asked each student in his class in turn to come to the board, select two of the numbers already present and write on the board their product (without erasing anything).  
Is it possible that at some moment number  $5000\dots 0$  (2023 zeros) appears on the board?
4. The bottom of a rectangular box is covered by tiles of sizes  $2 \times 2$  and  $1 \times 4$  (without overlaps) Sophia lost a  $2 \times 2$  tile and decided to replace it by a  $1 \times 4$  tile (she had a spare).  
Prove that now, she won't be able to cover the bottom of the box using these tiles.