ADVANCED MATH PROBLEM SOLVING CLUB
ASSGINEMNR 11: COOPERATIVE STRATEGIES

MARCH 5, 2023

General hint to many of the problems: if you have n different items (or items of n different kinds), replace them by remainders mod n. Then you can do things such as adding or multiplying the values.

Problems

1. A palace has 17 rooms, arranged in a singe line. One of the room is occupied by the princess, and the other 16, by her maids. A knight who is in love with the princess comes to the palace every night and knocks on the window, trying to find her. If he knocks on the window of the princess room, she runs away with him and they live happily together ever after. However, if she knocks on the window of the maid's room, she raises the alarm, and the knight has to leave – he can only come back the next night.

To make the matters worse for the knight, the king is moving the princess to a different room every night; however, he always moves her to a room next to the one she occupied the previous night.

Can the knight find the princess if he only has one month? Is there a strategy that guarantees it faster?

2. Two card players and a spectator are sitting around a table. A card deck is shuffled, and 7 cards are removed from it and shown to everyone. After that, these 7 cards are distributed among them in secret: 1 card to the spectator, 3 to each of the players.

The players can talk to each other, but the spectator will hear everything they say. Can they somehow exchange information so that at the end, each of the players will know what cards the other player has but the spectator doesn't know where any of the 6 cards (not including his own) is?

3. Black or white hats are placed on your and on mine heads. You see my hat, I see your hat, but none of us sees the hat on his own head. Each of us (without any sort of communications) must try to guess the color of his hat. When a signal is given each of us simultaneously says one word only: "black" or "white". We will win if and only if at least one of us has guessed correctly. Before this test we hold a consultation. How should we act in order to win in all possible situations?

4. Similar to the previous one, but now we have 2021 wizards and there are 2021 possible hat colors (each color may be used more than once, or not at all). Each wizard must try to guess the color of his own hat; if at least one of the guesses correctly, everyone wins. Is there a strategy that guarantees winning?

5. Alice and Bob are playing the following game. Each of them tosses a coin (usual fair two-sided coin) 1000000 times. After that, each of them records a number between 1-1000000 and gives it to the judge.

The judge reads these numbers (call them a for Alice, b for Bob) and then sees the result of the corresponding coin toss:

– Bob's a-th toss
– Alice's b-th toss

If these results are equal, they both win.

Can you find a strategy which gives Alice and Bob better than 1/2 winning chance?

As usual, they can agree on a strategy beforehand, but can not exchange any information once the game starts.

Hint is on next page.

[Hint: here is a simple strategy: each of them, if his/her first toss was heads, gives number 1, and otherwise, gives number 2. What is the probability of winning in this case?]