

## MATH CLUB: COLORING AND CUTTING

DEC 11, 2022

1. Brandon took a regular  $8 \times 8$  chessboard and cut out two squares: bottom left and upper right corners.  
Is it possible to cover the remaining board by  $2 \times 1$  rectangles without overlaps?
2. Is it possible to cut a  $10 \times 10$  board into  $1 \times 4$  pieces? (All cuts must follow the grid lines.)
3. On every square of a  $9 \times 9$  board there is an ant. At a signal, each ant moves to one of the squares diagonally adjacent to his. As a result, some squares will have more than one ant, and some will be empty.  
What is the smallest possible number of empty squares?
4. As you know, in chess a knight moves one square to one side and two squares in perpendicular direction.  
Is it possible for the knight to start in the corner of a  $7 \times 9$  board and visit each square of this board exactly once before returning to the starting position?  
What about  $4 \times 8$  board?
5. On a square ruled sheet of paper Sasha marked 2020 squares. Prove that it is possible to select 505 of them so that the selected squares have no common points (not even corners).
6. (a) Is it possible to cut a  $6 \times 6 \times 6$  cube into  $1 \times 2 \times 4$  pieces?  
(b) Is it possible to cut a  $3 \times 3 \times 3$  cube with center piece removed into  $1 \times 1 \times 2$  pieces?
7. Michael took a  $29 \times 29$  square ruled sheet of paper and cut out 99  $2 \times 2$  squares out of it (all cuts follow the grid lines). Show that the remaining part contains at least one more  $2 \times 2$  square.
8. The streets of city X form a triangular grid so that each city block is an equilateral triangle. Traffic laws require that at each intersection, you can either go straight or turn  $120^\circ$  (left or right).  
Two cars start from the same intersection in the same direction, but later take different routes. Can it be that they meet again in the middle of some street going in opposite directions?
9. The plane is colored using 3 colors: red, green, and blue. Prove that it is possible to find 2 points at distance 1 which have the same color.