ASSIGNMENT 1: MISCELLANEOUS PROBLEMS

SEPTEMBER 18, 2022

1. During daytime, a snail climbs 10cm up a post. During the night, it slides down 9cm. How long will it take the snail to reach the top of the pole if the height of the pole is 1m?

2. You have two identical glass balls. Your goal is to find the maximal height from which these balls can be dropped without breaking. To do that, you can drop the balls (one at a a time) from any floor of a 100-story building.

How many attempts will you need?

3. A traveler comes to an inn where he wants to stay for several days. The rate is 1 gram of gold per day. The traveler has a large gold bar and offers the innkeeper to pay for his stay by cutting from the bar a piece of appropriate weight. “No”, says the innkeeper, “you never know what happens tomorrow. I want to be paid every day for that day’s stay”.

“But this means that I have to cut you a one gram piece of gold every day”, says the traveler. “It is a lot of trouble”.

“Not necessarily”, says the innkeeper. “I can use the piece you give me today to give you change tomorrow — so you do not really have to cut it into one gram pieces”.

What is the minimal number of gold pieces the traveler has to cut if he is staying at the inn one week? one month (31 days)? Can you carefully explain why the answer suggested by you is indeed minimal possible?

4. A spider sits an a corner of a cubical room. Find the shortest path for it to reach the exactly opposite corner. Is such a path unique?

Note: spiders can’t fly.

5. Solve the equation

$$2022 - 2(2022 - 2(2022 - 2(2022 - 2x))) = x$$

6. A rectangular bar of chocolate consists of $m \times n$ squares. You want to break it into mn individual squares. At each step, you may pick up one piece you have and break it along any of the vertical or horizontal lines separating the squares.

How many breaks do you need? What is the fastest way to do it?

7. In each square of 9×9 board there is a pawn. Can you move each pawn to one of the adjacent squares so that after the move, there is again one pawn in each square of the board?

8. Prove that the number $30^{239} + 239^{30}$ can not be prime. [Hint: 31 is prime]

9. Kathryn has drawn on the blackboard a 4×6 grid. Nestor wants to erase some segments so that one can still reach any of the intersection points of the original grid, by starting at the corner and following the chalk lines. What is the maximal number of segments Nestor can erase? (Or, if it is easier: what is the smallest number of segments he can leave?)