Homework 5

Electric potential

The potential energy where of two charges separated by a distance *r* is

$$P = k\frac{q_1 \cdot q_2}{r} \quad (1)$$

Let us keep one of the charges, say, $\boldsymbol{q_1}$ fixed and change the charge $\boldsymbol{q_2}$. Since there is a product of the charge magnitudes in the numerator of formula (1), the potential energy will increase or decrease proportionally to the charge magnitude of $\boldsymbol{q_2}$. We can now calculate the potential energy *per unit charge*. For this we will divide the potential energy of the interacting charges $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ by the magnitude of $\boldsymbol{q_2}$:

$$\frac{P}{q_2} = k\frac{q_1 \cdot q_2}{r} \div q_2 = k\frac{q_1}{r} \quad (2)$$

We can imagine that each point of space around the charge q_1 can be characterized by the potential energy of a positive unit charge in this point. The electrostatic potential energy of a positive unit charge in a certain point is called "***electric potential"*** in this point. The electric potential is a scalar. The electric potential $\boldsymbol{\varphi}$ created by the point charge $\boldsymbol{Q}$ is:

$$\boldsymbol{\varphi = k\frac{Q}{r}} \quad (3)$$

There are 2 important issues. First: if the charge q is negative, the potential will be negative as well. Second: potential, created by a point charge is "spherically symmetrical". This means that only the distance from the charge to the point where we are measuring potential matters (rather than the exact position of the point).

The formula (3) means that a unit positive point charge placed at the distance r from the charge q will have potential energy $\boldsymbol{\varphi}$. If we will place an arbitrary charge Q at the distance r (instead of a unit charge) then the potential energy of the charge Q can be calculated as:

$$\boldsymbol{P = k\frac{Q}{r} \cdot q = \varphi \cdot q} \quad (4)$$

As we can see from the formula (3) the potential created by a point charge depends on the distance to the point charge. Difference of potentials taken in points A and B equals to the difference of potential energy of a unit positive charge in these points. Now let us look at Figure 1. Let us assume that the position of positive charge $\boldsymbol{Q}$ is fixed, and another small object having charge $\boldsymbol{q}$ is placed in a point separated from $\boldsymbol{Q}$ with a distance $\boldsymbol{R_1}$. Charge $\boldsymbol{q}$ is repelled by charge Q, and, being released, starts moving from charge $\boldsymbol{Q}$. When it reaches point B, separated from $\boldsymbol{Q}$ with a distance $\boldsymbol{R_2}$, it has nonzero kinetic energy, but its potential energy is less now. The gain in kinetic energy is equal to the loss of the potential one due to energy conservation.

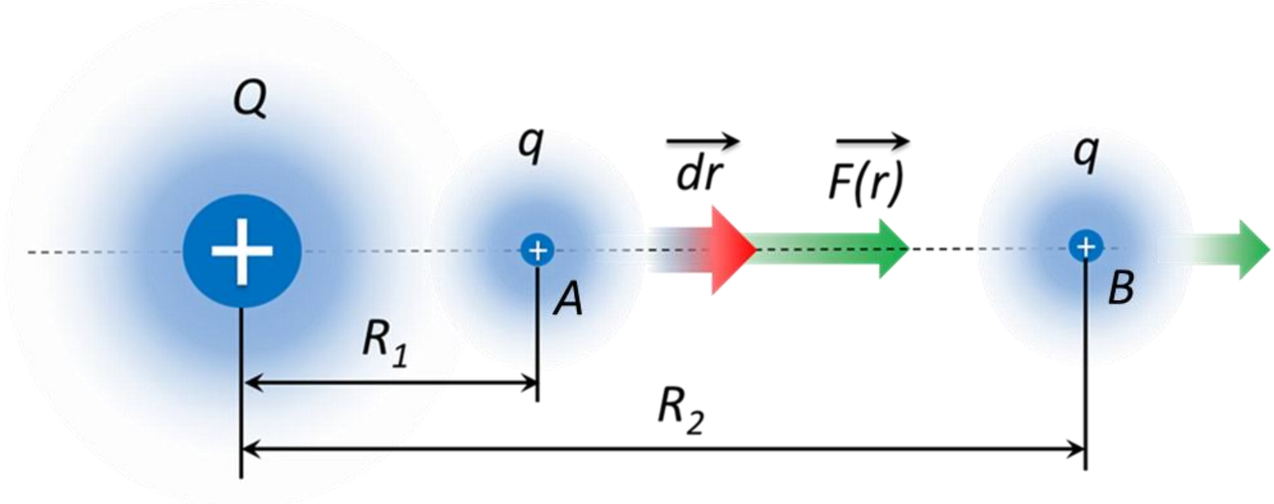


Figure 1. Work of the electric field. dr is a displacement in the direction AB, F(r) is the Coulomb force which depends on r.

But, the gain in kinetic energy is equal to the work $\boldsymbol{W_{AB}}$, done by electrostatic force on charge q. Typically, by "change" we mean "final value minus initial value" Change in potential energy then will be negative since its final valuer at $\boldsymbol{R_2}$ is less than its initial value at $\boldsymbol{R_1}$ (since $\boldsymbol{R_2}$ is larger than $\boldsymbol{R_1}$). Let us introduce a slightly different parameter: potential energy in the *initial* point minus potential energy in the *final* point. We have:

$$\boldsymbol{W_{AB} = P_A - P_B = q\varphi_A - q\varphi_B = q(\varphi_A - \varphi_B) = qU_{AB}} \qquad (5)$$

Here P_A, P_B –electrostatic potential energies in points A and B; $\boldsymbol{\varphi_A}$, $\boldsymbol{\varphi_B}$ – the electrostatic potentials, $U_{AB} = \boldsymbol{\varphi_A - \varphi_B}$ is the potential difference which is also called "***voltage drop** between points A and B"*, or just "***voltage** between points A and B"*.

We assumed that the charge was moved from point A to point B along a straight line. We may ask: "may be there is a an optimal path, so if we use this path the loss of potential energy when the charge moved from one point to the other will be minimal". This does not work for electrostatic field. **The potential energy (or just potential) difference between two points does not depend on the path we choose!**

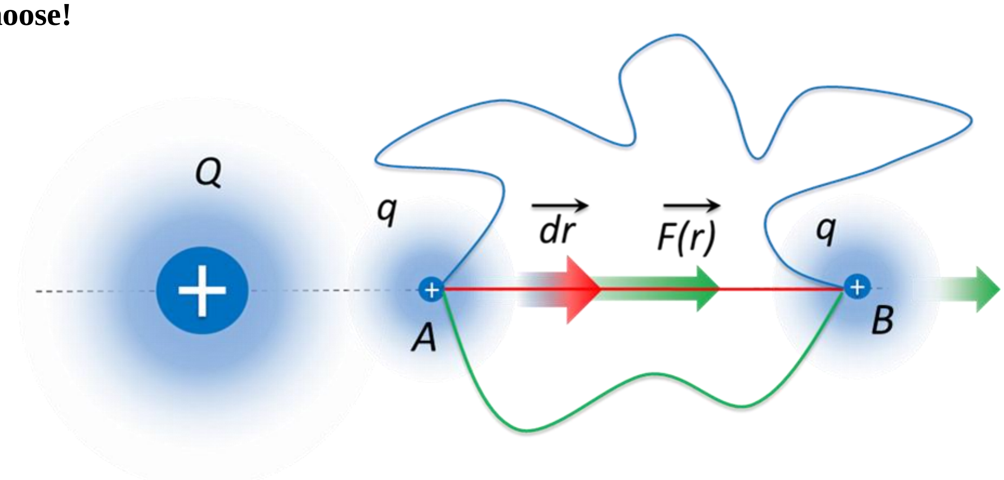
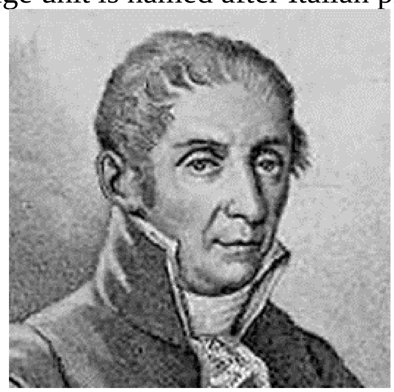


Figure 2. Potential energy difference between points A and B does not depend on the path geometry.

We can choose the green path (Figure 2) or the blue pass the potential energy difference will be the same. Let us assume for a moment that the change in potential energy depends on the path you choose and for the blue path it is, say, higher that for the green path. Then we could choose green path to go from A to B and blue pass to go from B to A and return to the same point A having higher potential energy than we initially had in this point. But this does not agree with the expression for electrostatic potential energy we obtained earlier. According to this expression if the distance is the same so is the potential energy. The electrostatic potential and voltage are measured in Volts.
1Volt=1Joule/1Coulomb. The voltage unit is named after Italian physicist Alessandro Volta:



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Problems:

1. An object with a charge of 0.01C being accelerated by electrostatic force moves from point A to point B and gains kinetic energy of 6J. Find the potential difference between points A and B.

2. There is a point charge of -1C (see picture below). The distance between the charge and the point A is 100m, the distance between the points A and B is also 100m. Find the potential difference between points A and B.