MATH 7 HOMEWORK 3: Algebraic operations continued
October 17, 2021

1. Exponents Laws

If a is a real number, n is a positive integer $$a^n = \underbrace{a \times a \times \cdots \times a}_{\text{n-times}}$$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

2. Radicals

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. Main Algebraic Identities

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

MATH 7 HOMEWORK 3: Algebraic operations continued
October 17, 2021

1. Simplify:

$$a)\ \frac{1}{(x+1)} - \frac{1}{(x-1)}$$

$$b)\ \left(1+\frac{1}{x}\right) \div (x+1)$$

$$c)\ \left(1+\frac{1}{x}\right) \div \left(1-\frac{1}{x}\right)$$

2. Simplify, use powers for radicals
   a. $\sqrt{\frac{56}{13}} \cdot \sqrt{\frac{26}{7}} =$
   b. $\sqrt[3]{\frac{56}{13}}\ \sqrt[3]{\frac{26}{7}} =$
   c. $\sqrt{48} =$
   d. $\sqrt[4]{48}$
   e. $\frac{\sqrt{48}}{\sqrt{15}} =$
   f. $\sqrt[3]{a^2\ b^3\ c^4 d^5}$
   g. $\sqrt[4]{a^2\ b^3\ c^4 d^5}$
   h. $\sqrt[5]{a^2\ b^3\ c^4 d^5}$

3. Express the following expressions in the form $2^r 3^s a^m b^n$:
   a. $8a^3 b^2 (27a^3)(2^5 ab) =$
   b. $3^2(2ab)^3(16a^2b^5)(24b^2a) =$
   c. $16a^2b^3(6ab^4)(ab^2)^3 =$

4. Expand as sums of powers of x:
   a. $(2x+5)^2 =$
   b. $(2-4x)^2 =$
   c. $(1-2x)^2 =$

5. Factor (i.e., write as a product) the following expressions:
   a. $4x^2 + 8xy + 4y^2$
   b. $9x^2 - 25$
   c. $(x-2)^2 - (y+3)^2$
   d. $256 - a^8b^8$
   e. $(x-2)^2 - 10(x-2) + 25$
   f. $3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$
   g. $a^2 - b^2 - 10b - 25$
   h. $x^4 + 4 \quad Hint: add\ and\ subtract\ 4x^2$

6. Solve the following equations. :
   a. $5(x+1) = 3x+2$
   b. $(x^2-1)(x+2) = 0$
   c. $\frac{x+2}{x+3} = 2$
   d. $(x-3)(x+4) = 0$
   e. $x^2 + 4x = 0$
   f. $x^3 + 4x = 0$

7. Prove:
   a. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   b. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   c. $Find (a+b)^4, (a-b)^4\ using\ the\ previous\ results$