

Math 6d: Homework 19

HW#19 is due March 10; submit to Google classroom 15 minutes before the class time.

Please, write clearly which problem you are solving and show all steps of your solution.

Basic probability

Basic probability rule, assuming all outcomes are equally likely:

$$P(\text{win}) = \frac{\text{number of winning outcomes}}{\text{total number of possible outcomes}}$$

Example: Probability of drawing a spade card out of the standard deck is $P(\text{spade}) = \frac{13}{52} = \frac{1}{4}$

Complement rule

If the probability of some event is P then the probability that this event will **not** happen is $1 - P$. For example, if we draw a card from the deck then the probability that it is not a spade is $1 - \frac{1}{4} = \frac{3}{4}$.

Product Rule

If we do two trials (e.g., rolling a die twice), then the probability of getting result A in the first trial and result B in the second one is

$$P(A, \text{ then } B) = P(A)P(B)$$

if the result of the second trial does not depend on the results of the first one.

Example: Tossing a coin

Question. If you toss a coin 10 times, what is the probability that all will be heads?

Answer. $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using a calculator, one can compute that this is $1/1024 \approx 0.001$, or 1/10th of 1%).

Question: If you toss a coin 10 times, what is the probability that all will be tails?

Answer. The same as above.

Question: If you toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer: Unfortunately, there are many combinations that give at least one head. In fact, it is easier to say what combinations do not give at least one head: there is exactly one such combination, all tails. The probability of obtaining this combination is, as we computed,

$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$. The remaining combinations will give at least one head; thus the probability of getting at least one head is $1 - \left(\frac{1}{1024}\right) = \frac{1023}{1024} \approx 0.999$.

Homework questions

- 1) You take the standard card deck and draw one card. What is the probability that the card will be
 - (a) Queen of hearts
 - (b) Either a queen or a hearts card
 - (c) A red card
 - (d) A picture card (a jack, queen, king, ace)
 - (e) A picture card other than the queen of hearts

- 2)
 - (a) What is the probability that when we toss a coin 4 times, there will be no heads?
 - (b) A and B are playing the following game. They toss a coin 4 times; if there are no heads, A wins and B pays him \$10. Otherwise, A loses and he pays \$1 to B. Would you prefer to play for A or B in this game?

- 3)
 - (a) What is the probability that when we roll two dice, at least one will be a 6?
 - (b) A and B are playing the following game. They roll two dice; if at least one is a 6, A wins and B pays her \$5. Otherwise, A loses and she pays \$1 to B. Would you prefer to play for A or B in this game?

- 4)
 - (a) What is the probability that if we roll 3 dice, all the numbers will be odd?
 - (b) A and B are playing the following game. They roll 3 dice; if all numbers are odd, A wins, and B pays him \$5. Otherwise, A loses and he pays \$1 to B. Would you prefer to play for A or B in this game?

- 5) Suppose there is an equal chance that a boy or a girl is born; what is the probability that the first five babies born next Saturday morning at St. Charles Hospital will be girls? That at least one of the five babies will be a girl?

- 6) In a certain club of 30 people, they are selecting a president, a vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?