Math 6d: Homework 15

HW#14 is due February 3; submit to Google classroom 15 minutes before the class time.

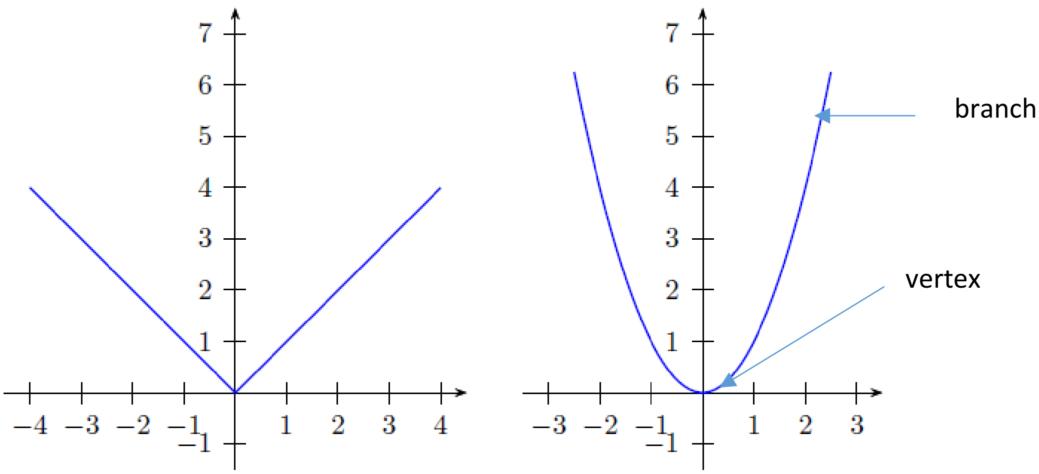
Please, write clearly which problem you are solving and show all steps of your solution.

Graphs

Generally, a graph of a function, $y = f(x)$, is a line in the $x - y$ plane. If one has two graphs $y = f(x)$ and $y = g(x)$ one can find **intersection** points of corresponding graphs by solving the system of equations. For example, the intersection point of two straight lines $y = x + 2$ and $y = -x$ is the point $(-1, 1)$ as x = −1 and y = 1 satisfy both of these equations; that is the point $(-1, 1)$ lies simultaneously on both straight lines.

Graphs of $y = |x|$ and $y = x^2$

The figures below show graphs of functions y = |x| and $y = x^2$ (a quadratic function in powers of x); the second graph is called a *parabola.*



The **standard form** of a parabola, $y = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$, is hard to immediately visualize and graph.
In its **vertex form,** the parabola's coefficients $\boldsymbol{a}, \boldsymbol{h},$ and $\boldsymbol{k}$ are directly related to the shape of the graph

$$y = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k} \text{ (vertex form), where } h = -\frac{b}{2a} \text{ and } k = -\frac{b^2-4ac}{4a}.$$

The graph of a parabola with nonzero a, k, h coefficients, compared to $y = x^2$, is vertically stretched by a factor of $\boldsymbol{a}$ (if $a < 0$, this means flipping it upside down and then stretching by |a|), and then its vertex is moved to point (h, k). In particular, the branches go up if $a > 0$ and down if $a < 0$.

You can **convert from standard to vertex form**. List the coefficients $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ from the standard form, then calculate $\boldsymbol{h}$ and $\boldsymbol{k}$ from the equations above, and after that re-write the graph equation into its vertex form $y = a(x - h)^2 + k$. For example, $y = x^2 + x$ can be converted into $y = (x + \frac{1}{2})^2 - \frac{1}{4}$

The parabola either intersects $y = 0$ $(x - axis)$ at two points, does not intersect it, or touches $y = 0$ at a single point. These intersecting points are known as **roots**. Correspondingly, the quadratic equation has two roots, no roots, or one root respectively. One can easily check that this corresponds to $D > 0$, $D < 0$ and $D = 0$ respectively, where the **determinant** $D = b^2 - 4ac$ is fond using the quadratic equation in a standard form.

Homework questions

***To draw a graph** of an equation, chose a set of points x and find the corresponding y values. Draw the points on a graph and use quadrille (square) paper. Connect with a line or a smooth curve.*

1. Find the equation of the line which passes through the point (3,4) and has a slope +2. (Hint: you only need to find the intercept and write $y = ax + b$)

2. Find the equation of the line through points (-2, 0) and (0,2).

3. Sketch the graph of the functions: $y = |x+1|$ and $y = -x + 0.25$. How many solutions do you think the following equation has?

$$|x+1| = -x + 0.25$$

   Note: you are not asked to solve the equation – just answer how many solutions there are.

4. Find the intersection point of a line $y = \frac{1}{4} x^2$ and a line $y = 2x+1$. Sketch or draw the graphs. (Hint: construct a system of equations and solve).

5. Sketch/draw graphs of the following functions (you may use desmos for this question). ***Then clearly describe the similarities and the differences between these graphs using full sentences.***
   a) $x + y = 2$ b) $y = |x-5| + 1$ c) $y = |x+1| + |x-2|$
   d) $y = |x+1| + |x+2| + |x+3|$

6. Sketch/draw graphs of the following function: $y = -x^2 + 4x - 3$
   a) To sketch, convert the function from standard to vertex form and use your knowledge of what the coefficients a, h, and k mean.
   b) If you cannot convert to vertex form, select x values for a few points, then calculate the corresponding y-values as you will do to graph any other function.
   c) Does the graph intersect the x-axis (when in the parabola's equation y is set to 0)? The intersecting points are known as **roots**.
   d) Does the number of roots correspond to the D-value? (*calculating the determinant is optional)