Math 6d: Homework 10

HW#10 is due December 9; submit to Google classroom 15 minutes before the class time.

Please, write clearly which problem you are solving and show all steps of your solution.

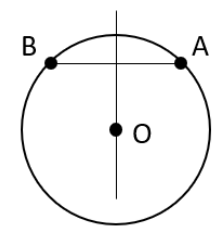
Summary from the classwork

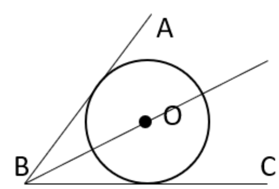
Paper folding: we discussed a different approach to geometric constructions: paper folding, or origami. Instead of using a ruler and a compass, we folded pieces of paper, starting with a square (or a rectangle). The attached pictures at the back show how one can construct various figures such as equilateral triangles.

And from before:

- **Operations** we can do using a ruler and compass.
  1. Construct the midpoint of a given segment *AB.*
  2. Construct the perpendicular bisector of segment *AB*, i.e. a line that goes through the midpoint of *AB* and is perpendicular to *AB*. (also a bisector to a chord on a circle)
  3. Given a line l and a point *A* on l, construct a perpendicular to the line l through *A*.
  4. Given a line l and a point *P* outside of l, construct a perpendicular to the line l through *P*.
  5. Given an angle *AOB*, construct the angle bisector (i.e., a ray *OM* such that $\angle AOM \cong \angle BOM$)

- **Important constructions and definitions**
  - ***Center of a circle:*** If two points *A*, *B* are on a circle, then the center of this circle lies on the perpendicular bisector to *AB* (i.e., a line that goes through the midpoint of *AB* and is perpendicular to *AB*).
  - ***Circle inscribed in an angle*** (circle inside in an angle). If a circle is inscribed in the angle *ABC*, then the center of this circle lies on the angle bisector.
  - ***Circle inscribed in a triangle*** (circle inside in a triangle). Construct 1) two angle bisectors that define the center of the circle and 2) a ⊥ line to one of the sides that passes through the center of the circle.
  - ***Circumscribed circle*** (circle around a triangle). Construct 1) two symmetry lines (bisectors) of two sides that define the center of the circle and 2) radius from the center of the circle to one of the triangle's vertices.



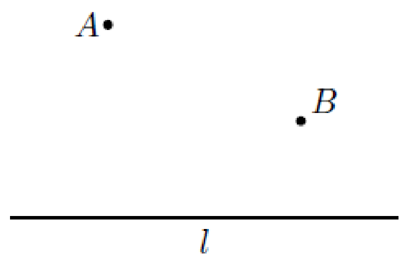


Homework questions

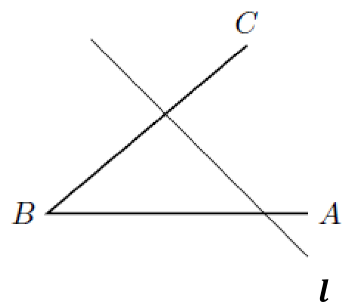
All of **the construction** problems should be done by only using a ruler and a compass. Do the constructions on a new sheet of paper and check the comments by William from your previous HW!

Construction problems 1, 2,3, and 4, start on the next page →

1. How can you find the point on the railroad (line ***l*** in the figure below) which would be at an equal distance from two villages (points A and B in the figure below)? [**Hint:** if this point is at an equal distance from A and B, then one can draw a circle with a center at this point which would go through A, and B...]. Draw the same figure in your homework (make it larger) and construct using a ruler and a compass. Do not draw on the homework page!

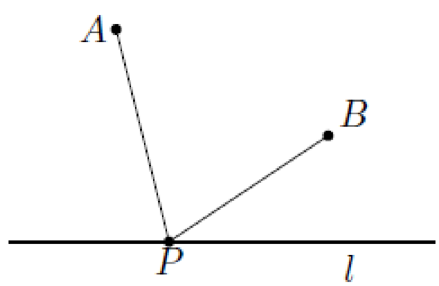


2. Given an angle ∠ABC and a line l intersecting both sides of this angle, find a point P on the line l which would be at an equal distance from the two sides of the angle (i.e., the two perpendiculars dropped from P to the sides of the angle would have the same length). Same here, re-draw the figure in your homework and construct.



3. Given a triangle, ABC, construct inside it a point that would be at an equal distance from all three vertices of the triangle. Draw a triangle (any triangle) in your homework and construct.

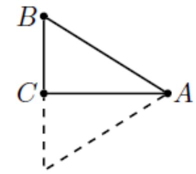
4. The figure below shows two villages A and B. A horseman starts at village A, goes to the river (line l in the figure) to let the horse drink, then goes to village B. How should he choose point P on the river to make his trip as short as possible?



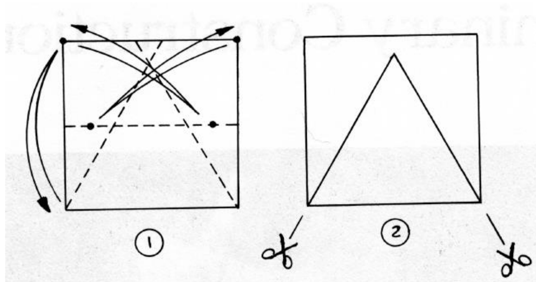
The **folding** problems 5, 6, and 7, start on the next page →

The following problems do not have to be submitted with the homework. Be sure to do them!

5. (a) Let ABC be a right triangle in which one of the legs is exactly 1/2 of the hypotenuse: BC = 1/2AB. What are the angles of such a triangle? (Hint: if you put two such triangles together, as indicated by the dotted line, what triangle do we get?)

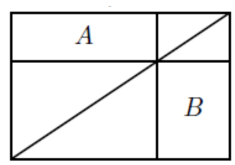


   b) The attached figure shows how you constructed in class an equilateral triangle from a square by folding. Can you explain why it does indeed give an equilateral triangle? Construct one again and check.



   c) How you can construct an equilateral triangle from a rectangle (by folding)?

6. The figure to the right shows a rectangle divided into several pieces. Which of the two rectangles, A or B, has a larger area? (Fold a rectangle and check!)



7. The attached figure shows how one can make a regular hexagon from a rectangular piece of paper (we did similar folding in class). Can you explain why this does give a regular hexagon? Make one and check that it is indeed regular.

(*funny double arrow below the first figure means "turn over and repeat step 1"*)