

**MATH 5: HANDOUT 17**  
**BEGINNING PROBABILITY – 2.**

PRODUCT RULE

**Question:** we roll two dice. What is the probability of rolling a 5 and a 6?

**Answer:** There are two ways of getting a 5 and a 6: as pair (5, 6) (5 on die number 1, 6 on die number 2) or as (6, 5) (6 on die number 1, 5 on die number 2). Thus, the answer is  $\frac{2}{36}$ .

**Question:** we roll two dice. What is the probability of getting sum of two numbers equal to 4?

**Answer:** there 3 ways of getting sum 4: (1, 3), (2, 2), (3, 1). Thus the probability is  $\frac{3}{36} = \frac{1}{12}$ .

**Question.** If toss a coin 10 times, what is the probability that all will be heads?

**Answer.**  $(\frac{1}{2})^{10} = \frac{1}{2^{10}}$  (using calculator, one can compute that it is  $1/1024 \approx 0.001$ , or 1/10 of 1%).

**Question.** If toss a coin 10 times, what is the probability that all will be tails?

**Answer.** The same.

**Question.** If we toss a coin 10 times, what is the probability that **at least one** will be heads?

**Answer.** Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed,  $1/2^{10} = \frac{1}{1024}$ . The remaining combinations will give at least one heads; thus probability of getting at least one heads is  $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$ .

PERCENTAGES AND FRACTIONS

So far we have mostly expressed probabilities as fractions. They can also be written as decimal numbers (between 0 and 1): for example,  $\frac{1}{5} = \frac{2}{10} = 0.2$ . It is also common to express probabilities as percentages: by definition,

$$1\% = \frac{1}{100} = 0.01$$

so  $x\% = \frac{x}{100}$ . For example,  $3\% = \frac{3}{100} = 0.03$ , and  $1.5\% = \frac{1.5}{100} = \frac{15}{1000} = 0.015$ .

This conversion is necessary when you multiply probabilities as the following example shows:

**Question.** The probability of winning in a certain game is  $p = 5\%$ . What is the probability of winning two times in a row?

**Answer.** According to multiplication rule it is  $p \times p = p^2$ . However, the answer  $5\% \times 5\% = 25\%$  is wrong. Correct answer is  $\frac{5}{100} \times \frac{5}{100} = \frac{25}{10,000} = 0.0025$ .

To convert from decimals to percent, multiply by 100:

$$p = (p \times 100)\%$$

For example,  $\frac{1}{5} = 0.2 = (0.2 \times 100)\% = 20\%$

## HOMework

1. If we roll two dice, what is the probability that the product of two numbers is a multiple of 3?
2. Recall that a roulette has 37 slots: 0 through 36. Among slots 1–36, half are red, the other half black (zero has no color). What is the probability of
  - (a) getting a red (on a single run of roulette)
  - (b) getting a red, then black, then 0 (on 3 successive runs)
  - (c) getting red 15 times in a row?
  - (d) getting this sequence of colors: RRRBRBRBBRBBRBR (also of length 15)?
3. A hunter is shooting ducks. Probability of hitting a duck with one shot is  $p = 1/3$ .
  - (a) What is the probability of missing the duck (with one shot)?
  - (b) He makes 5 shots. What is the probability that he misses all five?
  - (c) What is the probability that out of 5 shots, he will hit at least once? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
  - (d) What is the probability that out of 5 shots, he will hit exactly once? Will this probability double if he makes 10 shots?
  - \* (e) What is the probability that out of 5 shots, he will hit at least twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
  - \* (f) What is the probability that out of 5 shots, he will hit exactly twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
4. Supposing that there are equal chances of a boy or a girl being born, what is the probability that at least one of the first five babies born next Saturday morning at the St. Charles Hospital will be a girl? That all five will be girls?
5. At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win. Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
  - (a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
  - (b) Same questions for five balls.
  - (c) They charge you 2 dollars for 3 balls, or 3 dollars for 5 balls. Which is a better deal? [Considering only from the point of view of the chances of winning, not the fun you are getting]