

MATH 5: HANDOUT 5
ALGEBRAIC EXPRESSIONS AND FRACTIONS

DIVISION AND FRACTIONS

Today we discussed the last set of rules for arithmetic operations, involving division and fractions. The easiest way to remember them is to rewrite $a \div b$ in fraction form, as $\frac{a}{b}$. Then the rules are

$$\begin{aligned}\frac{a}{b} \times \frac{c}{d} &= \frac{ac}{bd} \\ \frac{a}{b} \div \frac{c}{d} &= \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \\ \frac{a+b}{c} &= \frac{a}{c} + \frac{b}{c} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}\end{aligned}$$

Example:

$$a \div (b \div c) = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$$

This can be used to solve equations. For example: to solve equation $\frac{5}{7}x = 15$, we multiply it by 7 to get $5x = 105$, then divide by 5 to get $x = 21$. Or we could solve it faster by multiplying both sides by $\frac{7}{5}$, to get

$$x = 15 \times \frac{7}{5} = \frac{15 \times 7}{5} = 21$$

HOMEWORK

1. Compute:

$$(a) 5 \div 7\frac{1}{2} \quad (b) (2 - 3.5) \times \frac{2}{3} \quad (c) 3\frac{2}{11} \div \frac{5}{7}$$

2. Simplify the following expressions

$$\begin{aligned}(a) x + 4(1 - x) \quad (b) 2 + 5x - 4(3 - x) \quad (c) 5(x - 1) - 3(2x + 1) \\ (d) x - \frac{4}{5}(1 - x) \quad (e) \left(\frac{2}{3}x + 1\right) \div \frac{6}{7} \quad (f) 2 \div (3 \div x - 1)\end{aligned}$$

3. Solve the following equations.

$$(a) \frac{3}{4}x = 12 \quad (b) \frac{1}{2}x + \frac{1}{7}x = 18 \quad (c) \frac{2}{3}(x + 7) = 12 \quad (d) 2.1x + \frac{1}{5}x + 1 = 9$$

4. John bought a large bag of red, green, and blue candies for Halloween, 74 candies in all. The number of red candies is one less than the number of green candies, and there are as many blue candies as red and green together. How many pieces of candy of each color are there?

5. A boy had a bag of apples. He gave $\frac{1}{2}$ of them to his parents, $\frac{1}{5}$ to his brother, $\frac{1}{4}$ to his sister and the last apple he ate himself. How many apples did he originally have?

6. Right now, Jane is 5 and her father is 42. In how many years will he be twice as old as she? Three times as old? (hint: denote the number of years by x and write the equation for x).

7. A hot water tap fills the bath in 5 minutes. The cold water tap fills the bath in 3 minutes. With both taps open, how long will it take to fill the bath?

8. A watermelon is 99% water (by weight). A watermelon jelly is 98% water (by weight). How much water does one have to evaporate from 1 kg watermelon to turn it into jelly?

*9. The numbers 1, 2, ..., 10 are written in a row. Is it possible to put signs + and - between them so that the result is equal to zero?