Math 4e. Classwork 14.

1. $a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

$$\frac{7^5}{7^3} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7} = \frac{7 \cdot 7 \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7}}{\cancel{7} \cdot \cancel{7} \cdot \cancel{7}} = 7 \cdot 7 = 7^2$$

$$\frac{7^5}{7^3} = 7^2 = 7^{5-3} = 7^{5+(-3)}$$

We can see that when we multiply (same) numbers in power, the exponent indices ("indices" is a plural of "index") should be summed. If we divide such numbers, the power of the divisor should be subtracted from power od dividend.

$$\frac{7^3}{7^4} = \frac{7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7} = \frac{1}{7} = 7^{3-4} = 7^{-1}; \qquad 7^{-1} = \frac{1}{7}$$

We can say that 7^{-1} is equal to 1/7, or to division of 1 by 7.

$$\frac{7^3}{7^6} = \frac{7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7} = \frac{1}{7 \cdot 7 \cdot 7} = 7^{3-6} = 7^3 \cdot 7^{-6} = 7^{-3} = \frac{1}{7^3}$$

Or, in a general way:

$$a^{-n} = \frac{1}{a^n} \qquad (a \neq 0)$$

Let's take a look on a few examples:

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$$

But also, we can write:

$$\left(\frac{2}{3}\right)^2 = \left(2 \cdot \frac{1}{3}\right)^2 = 2^2 \cdot \left(\frac{1}{3}\right)^2 = 2 \cdot 2 \cdot \frac{1}{3} \cdot \frac{1}{3} = 4 \cdot \frac{1}{9} = \frac{4}{9}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}; \qquad \left(\frac{1}{3}\right)^{-1} = \frac{1}{\frac{1}{3}} = 1 \cdot 3 = 3;$$

$$\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \frac{1}{\frac{3}{4} \cdot \frac{3}{4}} = \frac{1}{\frac{9}{16}} = \frac{16}{9} = \left(\frac{4}{3}\right)^2$$

We can write any number in the extended form using the power of 10:

$$5478 = 12000 \cdot 5 + 100 \cdot 4 + 10 \cdot 7 + 1 \cdot 8 = 10^3 \cdot 5 + 10^2 \cdot 4 + 10^1 \cdot 7 + 10^0 \cdot 8$$

We can write any number, big and small (decimal part can be extended with a concept of negative power:

$$7865.234 = 1000 \cdot 7 + 100 \cdot 8 + 10 \cdot 6 + 1 \cdot 5 + \frac{1}{10} \cdot 2 + \frac{1}{100} \cdot 3 + \frac{1}{1000} \cdot 4$$
$$= 10^3 \cdot 7 + 10^2 \cdot 8 + 10^1 \cdot 6 + 10^0 \cdot 5 + 10^{-1} \cdot 2 + 10^{-2} \cdot 3 + 10^{-3} \cdot 4$$

n	1	2	3	4	5	6
$\mathbf{10^n}$	10	100	1000	10000	100000	1000000

n	1	2	3	4	5	6	7	8	9	101
$\mathbf{2^n}$	2	4	8	16	32	64	128	256	512	024

Exercises:

1. Continue the sequence:

   $a.$ $1, 4, 9, 16$ ... $\qquad b.$ $1, 8, 27,$ ... $\qquad c.$ $1, 4, 8, 16$ ... $\qquad d.$ $1, 3, 9, 27$

2. What digits should be put instead of * to get true equality? How many solutions does each problem have?

   $a.$ $(2\,*)^2 = **\,1;$ $\qquad b.$ $(3\,*)^2 = ***\,6$ $\qquad c.$ $(7\,*)^2 = ***\,5$ $\qquad d.$ $(2\,*)^2 = **\,9$

3. Without doing calculations, prove that the following inequalities hold:

   Example:

   $39^2 < 2000$: $\quad 39 < 40, \quad 39^2 < 40^2 = 1600; \;\; 1600 < 2000.$

   $a.\; 29^2 < 1000; \quad b.\; 48^2 < 3000; \quad c.\; 42^2 > 1500; \quad d.\, 67^2 > 3500$

4. Represent numbers as a power of 10:

   Example: $1000^3 = (10^3)^3 = 10^{3\cdot 3} = 10^9$

   $100^2; \quad 100^3; \quad 100^4; \quad 100^5; \quad 100^6;$

5. Write the number which extended form is written below;

   Example: $2 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10 + 6 = 2726;$

   $a.\; 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 8;$ $\qquad b.\; 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 1;$

   $c.\; 9 \cdot 10^3 + 3 \cdot 10 + 3;$ $\qquad e.\; 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10 + 4;$

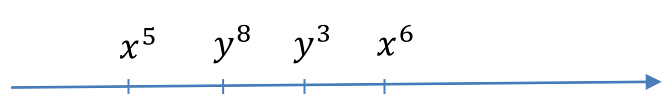
6. Compare:

   $a.\; 3^2 \quad 3^5;$ $\qquad b.\; (-3)^2 \quad (-3)^5;$ $\qquad c.\; (-3)^2 \quad (-3)^4$

   $d.\; \left(\frac{1}{3}\right)^2 \quad \left(\frac{1}{3}\right)^5;$ $\qquad b.\; \left(-\frac{1}{3}\right)^2 \quad \left(-\frac{1}{3}\right)^5;$ $\qquad c.\; \left(-\frac{1}{3}\right)^2 \quad \left(-\frac{1}{3}\right)^4$

7. $x^5 < y^8 < y^3 < x^6$

   Where 0 should be placed?



8. Represent as a fraction:

   Examples:

   $3^{-2} = \frac{1}{3^2} = \frac{1}{9};$ $\qquad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

   $a.\; 4^{-2}; \quad b.\; 3^{-3}; \quad c.\; 2^{-5}; \quad d.\; 5^{-2}$

9. A farmer has a cow, a goat and a goose. The cow and the goat will eat all the grass on his meadow in 45 days, the cow and the goose will eat all the grass on the same

meadow in 60 days, and the goat and the goose will eat all the grass on the meadow in 90 days. How many days will it take them altogether to eat all the grass on the meadow? (we assume that the new grass is not growing.)

10. Evaluate:

$$\left(1\frac{2}{5}+3.5:1\frac{1}{4}\right):2\frac{2}{5}+3.4:2\frac{1}{8}-0.35=$$

(Answer is 3. Write your solution.)