Math 4e. Classwork 13.

Work problems (combine labor problems).

Now let's solve some more problems.

Mary can eat her birthday cake in 10 minutes. Peter can eat the same cake in 15 minutes, how fast they will eat the same cake together?

These kinds of problems are related to the amount of work done per unit of time; we can call it "rate". To solve the problem, we have to find out what part of the cake Mary will eat in 1 minute. If she can eat the whole cake in 10 minutes, she only eats $\frac{1}{10}$ of the cake in one minute. Peter will eat $\frac{1}{15}$ of the cake in 1 minute. If they will start eating the cake simultaneously, each minute

$$\frac{1}{10}+\frac{1}{15}=\frac{3}{30}+\frac{2}{30}=\frac{5}{30}=\frac{1}{6}$$

will be eaten. We don't know, how many minutes are needed, but the rate with which the cake will be disappearing $\frac{1}{6}$ per minute:

$$x(minutes)\cdot\frac{1}{6}(part\ of\ the\ cake)=1(whole\ cake)$$

So, they will need exactly

$$x=1(whole\ cake):\frac{1}{6}(parts)=1\cdot 6=6\ minutes$$

Exponent.

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the base *a* and the exponent *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying *n* bases:

$$a^n=\underbrace{a\cdot a\cdot a\ldots\cdot a}_{n\ times}$$

In that case, a^n is called the *n*-th power of *a*, or *a* raised to the power *n*.
The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

If the same base raised to the different power and then multiplied:

$$4^3 \cdot 4^5 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^8 = 4^{3+5}$$

Or in a more general way:

$$a^n \cdot a^m = \underbrace{a \cdot a \ldots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \ldots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n+m\ times} = a^{n+m}$$

If the base raised to the power of *n* then raised again to the power of *m*:

$$(4^3)^5 = (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3)$$

$$= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$$

Or in a more general way:

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \ldots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times} \cdot \ldots \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

If we want to multiply $5^n = \underbrace{5 \cdot 5 \cdot 5 \ldots \cdot 5}_{n\ times}$ by another 5 we will get the following expression:

$$5^n \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n\ times} \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n+1\ times} = 5^{n+1} = 5^n \cdot 5^1$$

$$5^1 = 5$$

We can write the same property for any number:

$$a^n \cdot a = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \ldots \cdot a}_{n+1\ times} = a^{n+1} = a^n \cdot a^1$$

In order to have the set of power properties consistent, $a^1 = a$ for any number a.

Similarly, we can multiply any number by 1, this operation will not change the number, so if

$$a^n = a^n \cdot 1 = a^{n+0} = a^n \cdot a^0$$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number a, but 0.

Also, if there are two numbers a and b:

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{n\ times} = \underbrace{a \cdot \ldots \cdot a}_{n\ times} \cdot \underbrace{b \cdot \ldots \cdot b}_{n\ times} = a^n \cdot b^n \qquad (9)$$

Last property is

$$(a \cdot b)^n = a^n \cdot b^n$$

1. $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a,$ for any a
5. $a^0 = 1,$ for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exercises:

1. Mary, Peter, and Julia are going to do the spring clean up in their garden. Mary can do the job in 4 hours, Peter can do the full clean up in 3 hours, Julia need 6 hours to do the job. How fast they will do it together?

2. A swimming pool can be filled by pump A in 3 hours and by pump B in 6 hours, each pump working on its own. At 9 am pump A is started. At what time will the swimming pool be filled if pump B is started at 10 am?

3. The pool can be filled in 12 hours and emptied in 18 hours. Once, while the pool is being filled, the drain is accidentally left open. How long will it take to fill the pool from empty to full pool if both filling and draining pipe are opened?

4. The older brother can clean up the room in 2 hours, the younger brother can completely ruin it in 3 hours. In how many hours will the room be cleaned if they are locked together in the messy room? (it's a math problem, the answer "they will play games" will not be accepted!)

5. Three little pigs, Peter, Patty and Penny, decided to build a big house for all three of them. Peter and Patty can do it in 6 days, Patty and Penny can built the house in 8 days, and Penny and Peter can do the work in 12 days. How many days will take for them to build the house together?

6. Evaluate:

$$(-3)^3; \quad -3^3; \quad -3^3; \quad 2^7; \quad (-2)^7; \quad -2^7; \quad (2\cdot 3)^3; \quad 2\cdot 3^3; \quad \left(\frac{1}{3}\right)^2; \quad \frac{1}{3^2};$$

7. Write the following expressions in a shorter way replacing product with power:
Examples:
$(-a)\cdot(-a)\cdot(-a)\cdot(-a) = (-a)^4, \quad 3m\cdot m\cdot m\cdot 2k\cdot k\cdot k\cdot k = 6m^3k^4$

$a.\ (-y)\cdot(-y)\cdot(-y)\cdot(-y);$

$b.\ (-5m)(-5m)\cdot 2n\cdot 2n\cdot 2n;$

$c.\ -y\cdot y\cdot y\cdot y;$

$d.\ -5m\cdot m\cdot 2n\cdot n\cdot n;$

$e.\ (ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab);$

$f.\ p-q\cdot q\cdot q\cdot q\cdot q;$

$g.\ a\cdot b\cdot b\cdot b\cdot b\cdot b;$

$h.\ (p-q)\cdot(p-q)\cdot(p-q);$

8. Fill the table:

a	1	3	4	6	7	8	9	15
b	0	4	5	6	9	10	11	29
2a+2b								
2(a+b)								

9. 60 kids took part in the swimming meets. There were three times as many girls as boys. How many boys and how many girls competed? Write an equation and solve it.

10. Solve the equations:

$$a.\ 13\frac{2}{9}-\left(x+2\frac{5}{9}\right)=7\frac{5}{9};\qquad b.\ \left(y-4\frac{8}{11}\right)+1\frac{9}{11}=7\frac{3}{11}$$