Math 4d. Classwork 15.

There are 5 chairs and 5 kids in the room. In how many ways can kids sit on these chairs? The first kid can choose any chair. The second kid can choose any of the 4 remaining chairs, the third child has a choice between the three chairs, and so on. Therefore, there are $5 \times 4 \times 3 \times 2 \times 1$ ways how all of them can choose their places. Thus obtained long expression, $5 \times 4 \times 3 \times 2 \times 1$, can be written as 5!. By definition:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! \qquad or \quad n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 = n!$$

Write the following expressions as a factorial and vice versa:

Example: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$, $4! = 4 \times 3 \times 2 \times 1$

$10 \times 9 \times 8 \times \ldots \times 3 \times 2 \times 1 =$

$6! =$

$b \times (b-1) \times (b-2) \times \ldots \times 3 \times 2 \times 1 =$

$c! =$

Examples:

1. There are 20 students in the 4th grade. They have to choose a president, and a vice president of the class. How many different ways are there to do it?

There are 20 kinds to choose the president from, then there are only 19 possible ways to choose a vice president. So, the total number of the possible ways are $20 \cdot 19$.

2. There are 20 students in the 4th grade. They have to choose a team of two students to go to the math competition. How many different ways are there to do it?
   The problem seems to by very similar to the previous one, but there is a difference. In the problem number 3, if we chose Mary to be a president and John to be a vice-president and vice versa, we will have two completely different situation, and we count them as different way to have president and vice-president. And now, if we chose Mary and John to go to the math competition or John and Mary to go to the math competition, we will have the same team. So, the total number is $20 \cdot 19 : 2$

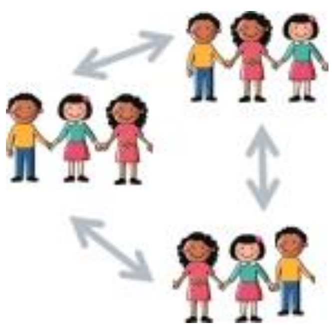
How many different 3-digit numbers can we create using 8 digits, 1, 2, 3, 4, 5, 6, 7, and 8 without repetition of the digits, i.e. such numbers that only contain different digits?

How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad?

What are the similarities in these two problems?

Can you see the difference between them?

In both cases, we have 8 possible ways to choose the first item (digit or student), 7 possible ways to choose the second item, and 6 different ways to choose the third one. So, there are $8 \cdot 7 \cdot 6$ different 3-digit numbers created from digits 1, 2, 3, 4, 5, 6, 7, and 8 and $8 \cdot 7 \cdot 6$ different teams of 3 students out of 8. Or not?



We can create numbers 123, 132, 213, 231, 321, 312 and they are all different numbers. If we chose Mike, Maria, and Jessika, a team of 3 students for the math Olympiad, it doesn't matter in which order we wrote their names.

In the first case, we have $8 \cdot 7 \cdot 6$ ways to create a 3-digit number out of 8 digits. In the second case for each group of 3 kids we will count 6 times (3! – number of ways to put 3 kids in line) more possible choices than there really are. So the total number of the way to choose the teem is

$$\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

Example:

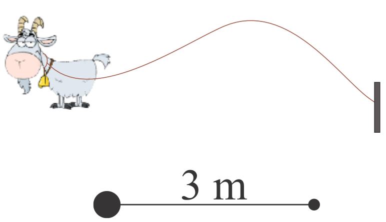
Mom has two apples, two pears, and a banana to gov to her child for lunch. How many different ways are there to do it during the week?(apples are identical, pears are identical).

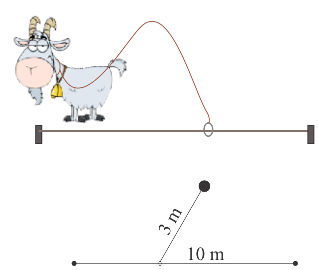
First, we can calculate the number of possible ways for five different fruits for 5 days of a school week. On Monday there are 5 possible options, on Tuesday there are 4, and so on… There are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ different ways. But she has 2 identical apples, so we are counting twice each possible combinations of fruits: when apple #1 goes first and apple #2 goes second and vice versa. But these situations are identical. Same goes for pears. The answer is

$$5!: 2: 2 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 5 \cdot 2 \cdot 3 = 30$$

Geometry.

Draw the picture of what shape will be left on the meadow? Use compass and ruler. Draw to scale1 cm for 1 m.





Exercises:

1. Simplify the following fractions:

   $a.\ \frac{5!}{7!};$ $b.\ \frac{n!}{(n-2)!};$

2. How many different ways are there to put 64 books on the shelf?

3. In the restaurant, there are 3 choices of starters, 4 choices of entrees and 5 choices of tasty desserts in the fix price dinner menu. How many different ways are there to fix a dinner for the restaurant's clients?

4. How many two-digit numbers can be composed from digits 1, 2, 3 without repetition of digits?

5. How many two-digit numbers can be composed from digits 1, 2, 3, if repetition is allowed?

6. Peter took 5 exams at the end of the year. Grade for exams are A, B, C, D. How many different ways are there to fill his report card?

7. There are red and green pencils in a box. How many pencils do you have to take out of the box without seeing them to be sure that you have at least 2 pencils of the same color?

8. If there are pencils of 5 different colors in a box, how many pencils do you have to take out to be sure that you have at least 2 of the same color? 3 of the same color?

9. There are 10 pairs of red gloves and 10 pairs of black gloves in a box. How many gloves do you have to take out to be sure that you have a pair of gloves that you can wear?

10. Write without parenthesis:

Example: $a - (a - b) = a - a + b = b$

$a.\ -(a-b);$ $b.\ -(c+d);$ $c.\ -(-x+y);$

$d.\ d-(-k+t);$ $e. -m+(a-c);$ $f.\ p-(-n+r-s);$

$g.\ c-(b+c-a)+(-a+b);$ $h.\ (d-m)-b-(-m+x+d)+x$

11. There are 80 students in three classes. There are 28 students in the first class, 6 students less in the second than in the third. How many students in the third class?

12. What will be the shape if the goat is attached to the frame like on picture below?