Math 4d. Classwork 14.

Exponent.

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the base *a* and the exponent *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying *n* bases:

$$a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$$

In that case, a^n is called the *n*-th power of *a*, or *a* raised to the power *n*.

The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3, 3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

If the same base raised to the different power and then multiplied:

$4^3 \cdot 4^5 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^8 = 4^{3+5}$

Or in a more general way:

$$a^n \cdot a^m = \underbrace{a \cdot a \ldots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \ldots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n+m\ times} = a^{n+m}$$

If the base raised to the power of *n* then raised again to the power of *m*:

$$(4^3)^5 = (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3)$$

$$= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$$

Or in a more general way:

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \ldots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times} \cdot \ldots \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

If we want to multiply $5^n = \underbrace{5 \cdot 5 \cdot 5 \ldots \cdot 5}_{n\ times}$ by another 5 we will get the following expression:

$$5^n \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n\ times} \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n+1\ times} = 5^{n+1} = 5^n \cdot 5^1$$

$$5^1 = 5$$

We can write the same property for any number:

$a^n \cdot a = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \ldots \cdot a}_{n+1\ times} = a^{n+1} = a^n \cdot a^1$

In order to have the set of power properties consistent, $a^1 = a$ for any number a.

Similarly, we can multiply any number by 1, this operation will not change the number, so if

$$a^n = a^n \cdot 1 = a^{n+0} = a^n \cdot a^0$$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number a, but 0.

Also, if there are two numbers a and b:

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{n\ times} = \underbrace{a \cdot \ldots \cdot a}_{n\ times} \cdot \underbrace{b \cdot \ldots \cdot b}_{n\ times} = a^n \cdot b^n \qquad (9)$$

Last property is

$(a \cdot b)^n = a^n \cdot b^n$

1. $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exercises:

1. Continue the sequence:

   $a.\ 1, 4, 9, 16 \ldots$ $b. 1, 8, 27, \ldots$ $c.\ 1, 4, 8, 16 \ldots$

2. What digits should be put instead of * to get true equality? How many solutions does each problem have?

   $a.\ (2\,*)^2 = **\,1;$ $b.\ (3\,*)^2 = ***\,6$ $c.\ (7\,*)^2 = ***\,5$ $d.\ (2\,*)^2 = **\,9$

3. Without doing calculations, prove that the following inequalities hold:

   Example:

   $39^2 < 2000:$ $39 < 40,$ $39^2 < 40^2 = 1600;$ $1600 < 2000.$

   $a.\ 29^2 < 1000;$ $b.\ 48^2 < 3000;$ $c.\ 42^2 > 1500;$ $d. 67^2 > 3500$

4. Evaluate:

   $(-3)^2;$ $-3^2;$ $(-3)^3;$ $2^7;$ $(-2)^7;$ $-2^7;$ $(2 \cdot 3)^3;$ $2 \cdot 3^3;$ $\left(\frac{1}{3}\right)^2;$ $\frac{1}{3^2};$

5. Represent numbers as a power of 10:

   Example: $1000^3 = (10^3)^3 = 10^{3 \cdot 3} = 10^9$

   $100^2;$ $100^3;$ $100^4;$ $100^5;$ $100^6;$

6. Write the number which extended form is written below;

   Example: $2 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10 + 6 = 2726;$

   $a.\ 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 8;$ $b.\ 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 1;$

   $c.\ 9 \cdot 10^3 + 3 \cdot 10 + 3;$ $e.\ 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10 + 4;$

7. Write the following expressions in a shorter way replacing product with power:

Examples:
$(-a)\cdot(-a)\cdot(-a)\cdot(-a)=(-a)^4$, $3m\cdot m\cdot m\cdot 2k\cdot k\cdot k\cdot k=6m^3k^4$

$a.\ (-y)\cdot(-y)\cdot(-y)\cdot(-y);$

$b.\ (-5m)(-5m)\cdot 2n\cdot 2n\cdot 2n;$

$c.\ -y\cdot y\cdot y\cdot y;$

$d.\ -5m\cdot m\cdot 2n\cdot n\cdot n;$

$e.\ (ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab);$

$f.\ p-q\cdot q\cdot q\cdot q\cdot q;$

$g.\ a\cdot b\cdot b\cdot b\cdot b\cdot b;$

$h.\ (p-q)\cdot(p-q)\cdot(p-q);$

8. A farmer has a cow, a goat and a goose. The cow and the goat will eat all the grass on his meadow in 45 days, the cow and the goose will eat all the grass on the same meadow in 60 days, and the goat and the goose will eat all the grass on the meadow in 90 days. How many days will it take them altogether to eat all the grass on the meadow? (we assume that the new grass is not growing.)

9. Evaluate:

$$\left(1\frac{2}{5}+3.5\div 1\frac{1}{4}\right)\div 2\frac{2}{5}+3.4\div 2\frac{1}{8}-0.35=$$

(Answer is 3) Write your solution.