

Math 4d. Classwork 5.

Algebra.

Multiplication and division of fractions.

Peter and 2 of his friends want to divide 2 chocolat bars evenly:

$$2 \div 3 = \frac{2}{3} = 2 \times \frac{1}{3}$$

You can see that the division by 3 (to finde $\frac{1}{3}$ of 2 chocolate bars) is the same as multiplacation by $\frac{1}{3}$. Also Peter has 18 candies. He wants to give to his friends $\frac{2}{3}$ of his candies. To find how many candies Peter should give to his friends he has to divide the number of candies by 3 and multiply by 2:

$$18 \div 3 \times 2 = 18 \times 2 \div 3 = \frac{18}{3} \times 2 = \frac{18 \times 2}{3} = 18 \times \frac{2}{3} = \frac{2}{3} \times 18$$

Fractions.

Multiplication of fraction.

What does it mean to multiply one fraction by another?

If we want to multiply a whole number by a fraction, we need to divide the number by denominator and multiply by nominator, as a result, we will get fraction part of our whole number.

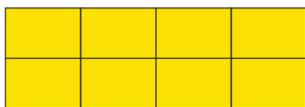
$$\frac{2}{3} \text{ of } 15 \text{ is } 15 \div 3 \times 2 = 10 = \frac{2}{3} \times 15$$

Now we want to find $\frac{1}{9}$ out of $\frac{1}{15}$.

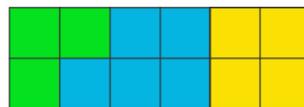
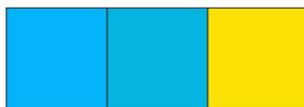
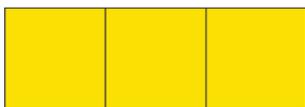
$\frac{1}{15}$ {						$\frac{1}{15} \times \frac{1}{15} = \frac{1}{15} \div 9 = \frac{1}{15 \times 9} = \frac{1}{135}$
						$\frac{2}{9} \times \frac{1}{15} = \frac{1}{15} \div 9 \times 2 = \frac{1 \times 2}{15 \times 9} = \frac{2}{135}$

More of fraction multiplication:

$$\frac{3}{8} \times \frac{2}{3} = \frac{3}{8} \div 3 \times 2 = \frac{3 \times 2}{8 \times 3} = \frac{2}{8} = \frac{1}{4}$$



$$\frac{2}{3} \times \frac{3}{8} = \frac{2}{3} \div 8 \times 3 = \frac{3 \times 2}{8 \times 3} = \frac{2}{8} = \frac{1}{4}$$



What does it mean that $\frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$? It really means that $\frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$ It is easy to see that

$$\frac{1}{4} \times \frac{8}{3} = \frac{1 \times 8}{4 \times 3} = \frac{2}{3}$$

And $\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$. So

$$\frac{1}{4} \times \frac{3}{2} = \frac{1 \times 3}{4 \times 2} = \frac{3}{8}$$

Let's try to look at this problem differently. If we are saying that $10 \div 2 = 5$ it means that 2 can fit into 10 five times (and $10 \div 5 = 2$ means that 5 can fit into 10 two times.)

Expression $\frac{3}{10} \div \frac{3}{5} = \frac{1}{2}$ means that $\frac{3}{5}$ can fit into $\frac{3}{10}$ only $\frac{1}{2}$ times.



We can see that to multiply 2 fractions one should multiply numerators and denominators. To divide one fraction by another we have to multiply first fraction by the fraction reciprocal to the second fraction. Two fractions are called reciprocal (or inverse) if their product is equal to 1.

$$\frac{3}{5} \times \frac{5}{3} = 1, \quad \frac{a}{b} \times \frac{b}{a} = 1$$

Geometry part is in the CW 2, you can read it there.

Exercises:

1. Compute:

a. $\frac{2}{3} - \frac{1}{4}$;

b. $\frac{7}{15} - \frac{1}{5}$;

c. $\frac{5}{12} + \frac{4}{15}$;

2. Simplify the following fractions:

$\frac{22}{66}$; $\frac{125}{75}$; $\frac{75}{100}$; $\frac{24}{360}$; $\frac{125}{1000}$; $\frac{100}{250}$; $\frac{198}{126}$

$\frac{2 \cdot 3}{4 \cdot 5}$; $\frac{2 \cdot 3}{7 \cdot 2}$; $\frac{5 \cdot 4}{4 \cdot 9}$; $\frac{7 \cdot 5}{2 \cdot 7}$

3. Evaluate:

a. $\frac{9 \cdot 4 \cdot 5}{20 \cdot 8 \cdot 15}$;

b. $\frac{9 \cdot 4 \cdot 5}{20 \cdot 8 \cdot 15}$;

c. $\frac{2}{3} \cdot \frac{6}{7} \cdot \frac{1}{10}$;

d. $\frac{2}{9} \cdot \frac{3}{7} \div \frac{5}{12}$;

e. $\frac{1}{3} \cdot 90 \cdot \frac{1}{5}$;

f. $\frac{9}{10} \div \frac{10}{11} \cdot \frac{100}{21}$;

g. $36 \cdot \frac{1}{12} \cdot \frac{1}{3}$

4. Evaluate:

a. $\frac{12 \times 5 + 12 \times 9}{12 \times 21}$;

b. $\frac{8 \times 8 + 8 \times 7}{8 \times 5}$;

c. $\frac{14 \times 5 + 14 \times 2}{28}$;

d. $\frac{19 \times 8 + 19 \times 6}{38} =$

5. There are three short stories in a book. Paulina read the first story in $\frac{1}{3}$ of one hour. She spent $\frac{1}{6}$ of an hour more reading the second story than reading the first one. The third story she read in $\frac{7}{12}$ of an hour less than the two previous stories together. How much time did it take her to read this book?

6. Compute by the most convenient way:

$$\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) + \left(\frac{2}{3} + \frac{2}{4} + \frac{2}{5}\right) + \left(\frac{3}{4} + \frac{3}{5}\right) + \frac{4}{5}.$$

7. Peter spent 2 hours doing his homework. $\frac{1}{3}$ of this time, he spent doing his math homework and $\frac{1}{4}$ of the remaining time he spent on the history assignment. How many minutes did Peter spend on his history assignment and how many minutes did he spend doing his math homework?

8. In a box there are blue, red, and green pencils. $\frac{2}{5}$ of the pencils are red, $\frac{1}{2}$ are blue. There are also 2 green pencils. How many pencils are in the box?

9. Julia's father's step is 70 cm long, Julia's step is 20 cm smaller. They start walking making their first step simultaneously. How far they should go to have next simultaneous step?

10. Draw two segments $[AB]$ and $[CD]$ in such way that their intersect

- by a point
- by a segment
- don't intersect at all.

11. Using a ruler draw a straight line, mark 3 points on the line, A , B , and C so that 2 rays are formed, BC and BA .

12. Draw three lines so, that they produce

- a. One point of intersection
- b. Two points of intersection
- c. Three points of intersections
- d. No intersections

(point of intersection is a point where two or more lines intersect)

13. Draw two rays \overrightarrow{AB} and \overrightarrow{CD} in such way that they intersect

- a. by a point
- b. by a segment
- c. by a ray
- d. don't intersect at all