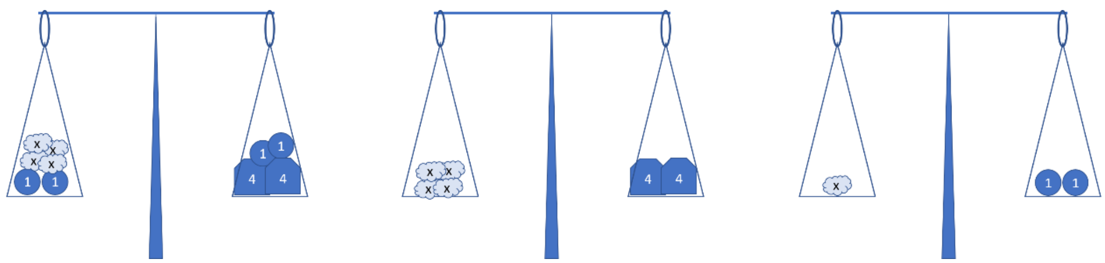
Classwork 12.

1. **Equation.**

What can we do with equations to solve them? Let’s take the last one.

$2+4x=10$. We can add and subtract the same quantity to (from) both sides of an equation, the balance will be in place. Also, we can multiply and divide both side by the same amount, kipping the balance.

$2+4x=10$ $\qquad$ $2-2+4x=10-2$ $\qquad$ $(4:4)x=8:4$



$2+4x=10$
$4x=10-2=8$
$x=8:4=2$

Equations are very useful to solve word problems. In each word problem there is an unknown quantity, and known parameters. The equation can be created with combinations of unknowns and known parameters. For example, let’s take a look on the following problem:

*There are 27 pencils in two boxes altogether. There are 5 more pencils In one box then in the other. How many pencils are there in each box?*

There are two unknown quantities in this problem, the number of pencils in the first box and the number of pencils in the second box. But these two quantities are not independent, one is 5 less than the other. If the number of pencils in one box is denoted as $x$, number of pencils in the second box will be $x+5$. And we also know that the total number is 27.

$$x+x+5=27$$

$$2x=27-5=22$$

$$x=22:2=11$$

Answer: there are 11 pencils in one box, and 16 in the other.

*There are candies in box. If each kid will take 4 candies, 19 candies will be left in the box. If each kid will take 5 candies, there will be lacking 2 candies. How many candies are there in the box?*

In this problem there are also two unknown quantities, the number of kids, and number of candies in the box. If the number of kids is denoted as $x$, the number of candies can be calculated in to ways:

First, $5 \cdot x - 2 =$ number of candies in the box

Second, $4 \cdot x + 19 =$ number of candies in the box, so

$$5 \cdot x - 2 = 4 \cdot x + 19$$

$$5x - 4x = 19 + 2$$

$$x = 21$$

The number of kids is 21. The number of candies can be calculated from either expression:

$5 \cdot 21 - 2 = 4 \cdot 21 + 19 = 103$

Answer: there are 103 candies in the box.

*There were 624 books in two boxes altogether. When $\frac{1}{3}$ of the books from one box and $\frac{3}{7}$ of the books from another box were sold to the customers, the number of books in each box became equal. How many books there were in each box at the beginning?*

In this problem there are two unknown variables, number of books in each box. Let's denote the number of books in the first box as $x$, and the number of books in the second box as $y$. Together $x + y = 624$. But we know also that

$$\frac{2}{3}x = \frac{4}{7}y$$

$$x = \frac{4}{7}y : \frac{2}{3} = \frac{4}{7}y \cdot \frac{3}{2} = \frac{4 \cdot 3}{7 \cdot 2}y = \frac{6}{7}y$$

We can now substitute $x$ in the equation $x + y = 624$ with $\frac{6}{7}y$.

$$\frac{6}{7}y + y = 624$$

$$\frac{13}{7}y = 624$$

$$y = 624 : \frac{13}{7} = 624 \cdot \frac{7}{13} = 48 \cdot 7 = 336$$

$$x = \frac{6}{7} \cdot 336 = 288$$

Answer: 288 books, and 336 books.

*On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?*

Again, there are two unknown amounts in the problem: number of yellow and number of white dandelions at the beginning, the sum of these two numbers is 35. We can use $y$ and $w$ as variable names for convenience.

$$y + w = 35$$

Which gives us the following relationship:

$$w = 35 - y$$

Also, we know that

$$2 \cdot (w - 8 + 2) = y - 2$$

$$2(w - 6) = y - 2$$

(eight whites are gone and two yellows are now white, and number of yellows now twice as big as number of whites). Using the substitution $w = 35 - y$, the last equation can be rewritten as

$$2(35 - y - 6) = y - 2$$

$$2(29 - y) = y - 2$$

$$58 - 2y = y - 2$$

$$58 + 2 = y + 2y$$

$$3y = 60$$

$$y = 20, \quad w = 35 - 20 = 15$$

Answer: at the beginning there were 15 white and 20 yellow dandelions.

1. Solve the equations:

   $a.\quad 5y + 3 = 10y - 12$ $\qquad b.\quad 3(2x + 3) = 27$ $\qquad c.\quad 5z - 20 = \frac{1}{3}(6x + 12)$

2. The sum of three consecutive odd numbers is 135. What is the smallest of the three numbers?

3. Mary bought 5 apples and 2 pears for \$4.60. Eva bought 8 apples and 6 pears for \$6.24. Veronica bought 3 apples and 3 pears. How much change did she get back from \$5.00?

**Inequalities**.

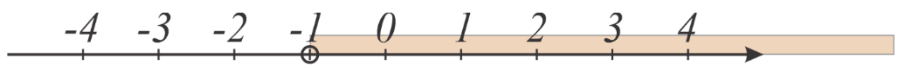
There is another type of problems, when we need ti find all possible values of varible which are greater (or smalle) than a particular number. In more sofisticated case, for which values of variable, one expression is greater (smaller) than another expression, for example:

$$x + 3 > 2x - 5$$

The simplest inequality is

$$x > a, \qquad x < a, \qquad where\ x\ is\ variable\ and\ a\ is\ a\ number.$$

$x > -1$,   the solution is all $x$, greater than 1,




Solution can be shown graphically are as $x\epsilon(-1, +\infty)$, or can be leaved as it is, it's already a solution (similarly as a solution of an equation $x = 2.$ )

We can add any number to both part of the inequality, the sign (< or >) will not change:

$x > -1$

$x + 2 > -1 + 2 \Rightarrow x + 2 > 1$

$y - 3 < 5$

$y - 3 + 3 < 5 + 3$

$y < 8, \qquad y \in (-\infty, 8)$

1. $x + 3 \ > -5$

Now let's try to multiply or divide both part of the inequality by the positive number.

If $x > 3$, then $2x$ will be grater then 6.

$x > 3\,, \qquad 2x > 6$

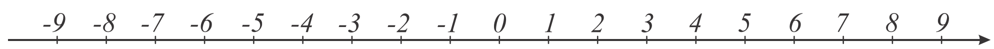
If $x > 3$ what can we tell about $-x$ ?

$-x \qquad 3 \cdot (-1)$

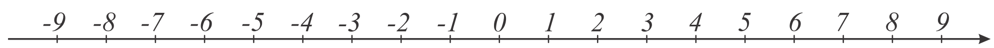
2. $x + 3 > 5x - 5$
3. $4\mathrm{x} - 3 \neq 0$
4. $3(\mathrm{x} - 1) < 5\mathrm{x} + 9$
5. $2x - 1 > - x + 3$
6. $|x| > 8$

7. Show on the number line points that are satisfying the following inequalities:

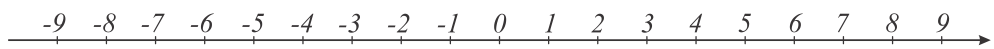
a) $|x| < 4$



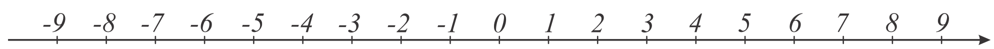

b) $|x| > 3$




c) $\left|x - \frac{1}{2}\right| > 3$




d) $\left|x - \frac{1}{2}\right| < 8$




8. $M = \{x|\ x > 5\},\ K = \{x|x < 20\}$

$M \cap K =$

$M \cup K =$

9. $M = \{x|\ x \leq 5\},\ K = \{x|x \geq 20\}$

$M \cap K =$

$M \cup K =$

10. Points $a$, 0, and $b$ are marked on the number line below:

Which of the following expressions is true?

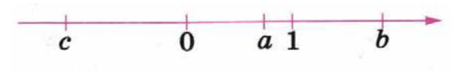
1) $a + b > 0$ or $a + b < 0$

2) $a - b > 0$ or $a - b < 0$

3) $ab > 0$ or $ab < 0$

4) $\frac{b}{a} > 1$ or $\frac{b}{a} < 1$

11. Points *a*, *b*, *c*, 0, and 1 are marked on the number line below:




Which of the following expressions is true?

1) $ab < b \text{ or } ab > b$
2) $abc < a \text{ or } abc > a$
3) $-ac < c \text{ or } -ac > c$

12. Prove that values of the following expressions do not depend from the value of variables. Find these values.
Hint: simplify these expressions.

$a.\quad \dfrac{4^m + 4^m + 4^m + 4^m}{4^m : 4^2}$ $\qquad b.\quad \dfrac{\overbrace{10^n + 10^n + 10^n + \ldots + 10^n}^{10\ times}}{10^n : 10}$

$c.\quad \dfrac{\overbrace{99^k + 99^k + 99^k + \ldots + 99^k}^{99\ times}}{99^k : 99}$

13. Simplify:

$a.\quad -(a + b)(a + b)$ $\qquad b.\quad -(x - y)(x - y)$

$c.\quad (a - b - c)(a - 1)$ $\qquad d.\quad (2m - n)(n - 3m)$

$e.\quad (a + 1)(a + 1)(a + 1)$ $\qquad f.\quad (x + 1)(x^2 - x + 1)$

14. Evaluate:

$$\frac{\left(5\frac{4}{45}-4\frac{1}{6}\right)\div 5\frac{8}{15}}{\left(4\frac{2}{3}+0.75\right)\cdot 3\frac{9}{13}}\cdot 34\frac{2}{7}+\frac{0.3\div 0.01}{70}+\frac{2}{7};$$

Rational number is a number which can be represented as a ratio of two integers:

$$a=\frac{p}{q}; \qquad p \in Z, and\ q \in N, \qquad (Z=\{\pm\cdots,\pm 1,0\}, N=\{1,2,\ldots\})$$

Rational numbers can be represented as infinite periodical decimals (in the case of denominators containing only powers of 2 and 5 the periodical bloc of such decimal is 0).
Numbers, which can't be express as a ratio (fraction) $\frac{p}{q}$ for any integers p and q  are irrational numbers. Their decimal expansion is not finite, and not periodical.
Examples:
0.01001000100001000001…
0.12345678910111213141516171819202 1…

What side the square with the area of $a$ m$^2$ does have? To solve this problem, we have to find the number, which gives us $a$ as its square. In other words, we have to solve the equation

$$x^2=a$$

This equation can be solved (has a real number solution) only if $a$ is nonnegative (($a \geq 0$) number. It can be seen very easily;

If $x=0, \quad x\cdot x=x^2=a=0,$

If $x>0, \quad x\cdot x=x^2=a>0,$

If $x<0, \quad x\cdot x=x^2=a>0,$

We can see that the square of any real number is a nonnegative number, or there is no such real number that has negative square.

**Square root** of a (real nonnegative) number $a$ is a number, square of which is equal to $a$.

There are only 2 square roots from any positive number, they are equal by absolute value, but have opposite signs. The square root from 0 is 0, there is no any real square root from negative real number.

Examples:

1. Find square roots of 16: 4 and $(-4)$, $4^2 = (-4)^2 = 16$
2. Numbers $\frac{1}{7}$ and $\left(-\frac{1}{7}\right)$ are square roots of $\frac{1}{49}$, because $\frac{1}{7} \cdot \frac{1}{7} = \left(-\frac{1}{7}\right) \cdot \left(-\frac{1}{7}\right) = \frac{1}{49}$
3. Numbers $\frac{5}{3}$ and $\left(-\frac{5}{3}\right)$ are square roots of $\frac{25}{9}$, because $\left(\frac{5}{3}\right)^2 = \frac{5}{3} \cdot \frac{5}{3} = \left(-\frac{5}{3}\right)^2 = \left(-\frac{5}{3}\right) \cdot \left(-\frac{5}{3}\right) = \frac{25}{9}$

**Arithmetic square root** of a (real nonnegative) number $a$ is a nonnegative number, square of which is equal to $a$.

There is a special sign for the arithmetic square root of a number $a$: $\sqrt{a}$.

Examples;

1. $\sqrt{25} = 5$, it means that arithmetic square root of 25 is 5, as a nonnegative number, square of which is 25. Square roots of 25 are 5 and $(-5)$, or $\pm\sqrt{25} = \pm 5$
2. Square roots of 121 are 11 and $(-11)$, or $\pm\sqrt{121} = \pm 11$
3. Square roots of 2 are $\pm\sqrt{2}$.
4. A few more:

   $\sqrt{0} = 0;$ $\quad \sqrt{1} = 1;$ $\quad \sqrt{4} = 2;$ $\quad \sqrt{9} = 3;$ $\quad \sqrt{16} = 4;$

   $\sqrt{25} = 5;$ $\quad \sqrt{\frac{1}{64}} = \frac{1}{8};$ $\quad \sqrt{\frac{36}{25}} = \frac{6}{5}$

Base on the definition of arithmetic square root we can right

$$\left(\sqrt{a}\right)^2 = a$$

To keep our system of exponent properties consistent let's try to substitute $\sqrt{a} = a^k$. Therefore,

$$\left(\sqrt{a}\right)^2 = (a^k)^2 = a^1$$

But we know that

$$(a^k)^2 = a^{2k} = a^1 \quad \Rightarrow \quad 2k = 1, \; k = \frac{1}{2}$$

And we can agree to consider arithmetic square root as fractional exponent

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\left(\sqrt{a}\right)^2 = a, \quad \left(\sqrt{b}\right)^2 = b, \quad \left(\sqrt{a}\right)^2\left(\sqrt{b}\right)^2 = \left(\sqrt{a}\sqrt{b}\right)^2 = ab = \left(\sqrt{ab}\right)^2 \Rightarrow \sqrt{a}\sqrt{b} = \sqrt{ab}$$

To solve equation $x^2 = 23$ we have to find two sq. root of 23. $x = \pm\sqrt{23}$. 23 is not a perfect square as 4, 9, 16, 25, 36 …