Classwork 7.

Algebra.

Monomials.

Monomial is a product of variables in nonnegative integer power and a number, which is called a coefficient. For example: $xy^3 6,\ 56,\ 3c^5\ d^{10}, 2x3y5$.
Two monomials are equal if their difference is only on the order of factors.
Monomial is equal to 0 if one of the factors is 0.
Usually, monomials are written in the following form: first goes a coefficient (only one number), then the variable with the highest power and so on…Example above $6y^3x, 56, 3d^{10}c^5, 30xy.$
Degree of a monomial is the sum of all exponents of variables. The degree of $6y^3x$ is 4 $(1 + 3 = 4)$.
Several monomials can be added together and/or multiply.

$$5x^2m^3 \cdot 7m^2y^3 = 35x^2m^5y^3$$

Polynomials.

We can add together a few monomials and get a polynomial:

$$A = 6y^3x + 56 + 3d^{10}c^5 + 30xy$$

The degree of a polynomial is the highest of the degrees of its monomials (individual terms) with non-zero coefficients. The term order has been used as a synonym of degree but, nowadays, may refer to several other concepts. For example, the polynomial $7x^2y^3 + 4x - 9$ which can also be expressed as $7x^2y^3 + 4x^1y^0 - 9x^0y^0$ has three terms. The first term has a degree of 5 (the sum of the powers 2 and 3), the second term has a degree of 1, and the last term has a degree of 0. Therefore, the polynomial has a degree of 5, which is the highest degree of any term.

To determine the degree of a polynomial that is not in standard form (for example: $(x + 1)^2 - (x - 1)^2$, one has to put it first in standard form by expanding the products (by distributivity) and combining the like terms; $(x + 1)^2 - (x - 1)^2 = 4x$ is of degree 1, even though each summand has degree 2.
Polynomials can be added together

$$(7x^2y^3 + 4xy^2 - 6) + (3x^2y^3 - 2x^5y^2) = 7x^2y^3 + 4xy^2 - 6 + 3x^2y^3 - 2x^5y^2$$
$$= 10x^2y^3 + 4xy^2 - 2x^5y^2 = -2x^5y^2 + 10x^2y^3 + 4xy^2$$

Multiplication of polynomials.

How to multiply polynomials?

$$(a + b) \cdot (c + d) =?$$

We know how to multiply an expression by a number using the distributive property: $a \cdot (b + c) = ab + ac$. What should we do to multiply one expression by another? To simplify the problem let's do the substitution, $a + b = u$ and use the distributive property:

$$(a + b) \cdot (c + d) = u(c + d) = uc + ud$$

This new expression is not exactly the result what we are looking for; so, we need to put back $(a + b)$ instead of u:

$$uc + ud = (a + b)c + (a + b)d$$

To get the final result let's use the distributive property again:

$$(a + b)c + (a + b)d = ac + bc + ad + bd$$

More polynomial multiplications:

$$(2x^2 + 3y^3) \cdot (3x^3 + y^5) = 2x^2 \cdot 3x^3 + 2x^2 \cdot y^5 + 3y^3 \cdot 3x^3 + 3x^3 \cdot y^5$$
$$= 6x^5 + 2x^2y^5 + 9x^3y^3 + 3x^3y^5$$

Algebraic identities are an expression which is true for any values of variables.

A few similar identities are very useful:

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

We already know what is GCD and LCM for several natural numbers and we know how to find them.

Find GCD (GCF) and LCM for numbers

a. 222 and 345.
b. $2^2 \cdot 3^3 \cdot 5$ and $2 \cdot 3^2 \cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:

$2x^2y^5 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y,$ $\qquad$ $4x^3y^3 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot$

$A = (Factors, 2x^2y^5) = \{2, x, x, y, y, y, y, y\}, B = (Factors, 4x^3y^3) = \{\ 2, 2, x, x, x, y, y\}$

Common devisors are any product of $A \cap B = \{\ 2, x, x, y, y\}$.

What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A \cup B = \{2,2, x, x, x, y, y, y, y, y\}$$

$$\frac{2x^2y^5}{2\cdot x^2y^2}=y^3; \qquad \frac{4x^3y^2}{2\cdot x^2y^2}=2x;$$

$$\frac{4x^3y^5}{2\cdot x^2y^5}=2x; \qquad \frac{4x^3y^5}{4\cdot x^3y^3}=y^2;$$

Algebraic fraction are expression are a fraction $\frac{A}{B}$ $(B \neq 0)$ whose numerator and denominator are algebraic expressions (not necessarily polynomials). For example:

$$\frac{3x^2+y}{y^2-5x+2}; \qquad \frac{\frac{1}{x}-3}{y+\frac{1}{y}}$$

$$\frac{A}{1}=A; \qquad \frac{A}{B}=\frac{A\cdot C}{B\cdot C} \quad (C\neq 0); \qquad -\frac{A}{B}=\frac{-A}{B}=\frac{B}{-A}$$

Exercises.

1. Which of the following are monomials?

a) $a;$	d) $a+b;$	g) $ba;$	j) $b2c;$
b) $\frac{ab}{a+b};$	e) $\frac{ax}{b};$	h) $\frac{3}{4}xy;$	k) $7a-3;$
c) $-1,(26);$	f) $(a-b)\cdot 3;$	i) $\frac{p}{b}axy;$	l) $0?$

2. Simplify the following expression (combine like terms, think about which terms you can add together and which you can't):

$$\left(\frac{1}{7}klm^2-\frac{4}{3}kl^2m+7klm\right)+\left(-\frac{3}{21}klm^2+\frac{4}{9}kl^2m-5klm\right);$$

3. Simplify the following expressions (rewrite the expressions without parenthesis, combine like terms);

Example:

$$(2x+3)\cdot(x+7)=2xx+2x\cdot 7+3x+3\cdot 7=2x^2+10x+21$$

$$(x+5)(x+y+3);$$

out

$(k-1+d)(k-d);$

$\frac{2}{3}+2x\left(\frac{1}{2}-\frac{1}{3}y\right)-x-\frac{1}{3}(2-2xy);$

$2x^2(x+y)-3x^2(x-y);$

4. Factor the common factor (factorize):

   $a.\ \ a^2+ab;$ $b.\ \ 2xy-x^3;$ $c.\ \ x^2y^2+y^4;$

   $d.\ \ x^2-x;$ $e.\ \ b^3-b^2;$ $f.\ \ a^4+a^3b;$

   $g.\ 4\,a^6-2a^3b;$ $h.\ \ a+a^2;$ $f.\ \ 9x^4-12x^2y^4;$

5. Evaluate:

$$\frac{(999^{-1}-1000^{-1})(999^{-1}+1000^{-1})}{(1000^{-1}-999^{-1})^2}$$

6. Simplify the following expressions (combine like terms):

   $a.\ \ 7a+(2a+3b);$ $b.\ \ 9x+(2y-5x);$
   $c.\ \ (5x+7a)+4x;$ $d.\ \ (5x-7a)+5a;$
   $e.\ \ (3x-6y)-4y;$ $f.\ \ (2a+5b)-7b;$
   $g.\ \ 3m-(5n+2m);$ $h.\ \ 6p-(5p-3a);$

7.

   $a.\ \ (x^2+4x)+(x^2-x+1)-(x^2-x);$
   $b.\ \ (a^5+5a^2+3a-a)-(a^3-3a^2+a);$
   $c.\ \ (x^2-3x+2)-(-2x-3);$
   $d.\ \ (abc+1)+(-1-abc);$
   .

8. Factorize the following polynomials:

   $a.\ \ x(1+b)+y(1+b);$ $f.\ \ (a+b)a-b(a+b);$
   $b.\ \ m(2k-3)+2(2k-3);$ $g.\ \ (x+y)3-a(x+y);$
   $c.\ \ 2a(1-b)-3(1-b);$ $h.\ \ a(b+3)-b(3+b);$
   $d.\ \ 7x(x-2y)-2(2y+x);$ $i.\ \ a(a+b)+(a+b);$
   $e.\ \ 2x(x-2y)+3y(x+2y);$ $j.\ \ 2x(a-1)-(a-1);$

9. Add fractions:

   Example:

$$\frac{2}{x^2a}+\frac{3}{a^2x}=\frac{2a}{a^2x^2}+\frac{3x}{a^2x^2}=\frac{2a+3x}{a^2x^2}$$

$a.\ \frac{1}{a}+\frac{1}{b};$ $\quad b.\ \frac{2}{x}-\frac{3}{y};$ $\quad c.\frac{x}{a}+\frac{y}{b};$

$d.\ \frac{5a}{7}-\frac{b}{x};$ $\quad b.\ \frac{1}{2a}-\frac{1}{3};$ $\quad c.\frac{1}{a}-\frac{1}{bc};$

10. Euler formula for prime numbers:
$n^2 - n + 41$ is a prime number for any $n \in N$. Prove or disapprove it.

11. Compute:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365}$$