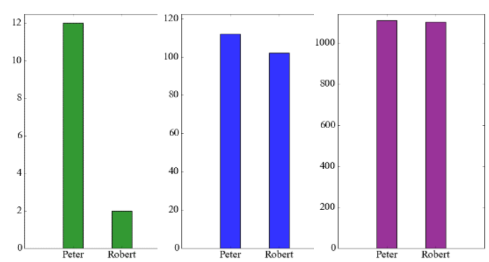
Classwork 6. Algebra.

Algebra.

Ratio and percent.

Peter has 10 dollars more than Robert. Is this a big difference? How we can compare the amount of money they have?
Take a look at the table

Peter	\$12	\$112	\$1112
Robert	\$2	\$101	\$1102



In all these cases the absolute difference is the same, but in the first situation Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are $\frac{12}{2}$; $\frac{112}{102}$; $\frac{1112}{1102}$;

The quantitative relation between two amounts showing the number of times one value contains or is contained within the other is called **a ratio**. The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

The ratio of two numbers indicates how many times one number is larger than another or which part of one number the other number is.

Three brothers, 5, 7, and 9 years old went to trick-o-treat. They got 84 sweets altogether. They decided to divide the candies in the ratio of their age 9:7:5. How many candies each of should get?
Two divide all candies between the brothers we need to find the "unit" part of the total amount of candies. The oldest brother should get 9 of such units, the middle one should get 7, and the youngest brother will get 5. Total amount of units is $9 + 7 + 5 = 21$. The number of candies is 84, so the "unit" contains $84: 21 = 4$ candies.

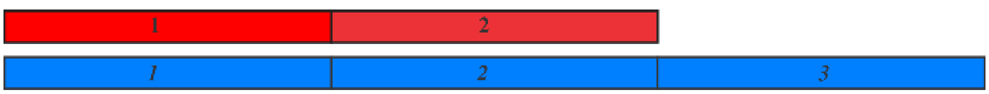
So, the first brother will get $4 \cdot 9 = 36$ candies, the second will gets $4 \cdot 7 = 28,$ and the third will get $4 \cdot 5 = 20.$ $36 + 28 + 20 = 88$

We can write the ratio of two numbers in the several ways:

$$a \text{ to } b, \qquad a{:}\,b, \qquad \frac{a}{b}$$

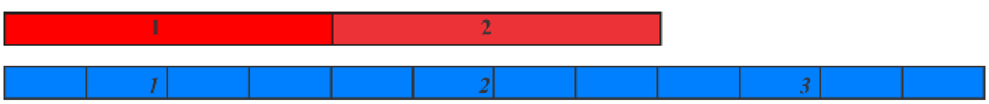
Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)

Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.



Step 3. Let's compare the number of sold and leftover balloons.

922
$1686 - 922 = 764$

Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations? Let's try.

$G + B + R = 1686$

$$3R = 2B$$

$$\frac{1}{2}G + R + \frac{1}{4}B = 922$$

$$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$$

$$R - \frac{1}{2}B = 158$$

$$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$$

1 percent of quantity is a $\frac{1}{100}$ th part of it.

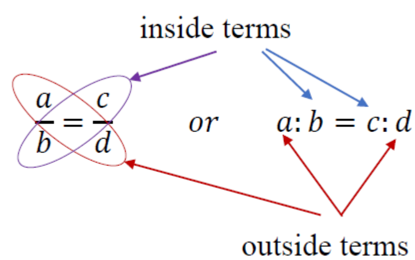
One percent (**1%**) means 1 per 100.

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.

Proportions have several interesting features.



1. The product of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ac}=\frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b}=\frac{c}{a}$$

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad a\cdot d=b\cdot c \quad \Leftrightarrow \quad \frac{ad}{ab}=\frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c}=\frac{b}{a}$$

4. Two inside terms can be switched as well.

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c}=\frac{b}{d}$$

5. Also, several other new proportion can be created.

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a\pm b}{b}=\frac{c\pm d}{d}$$

(the sign $\pm$ is used to show that both, addition and subtraction, can be used)

Let's prove one of the statements:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b}+1=\frac{c}{d}+1 \quad \Leftrightarrow \quad \frac{a}{b}+\frac{b}{b}=\frac{c}{d}+\frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b}=\frac{c+d}{d}$$

6. Another proportion:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d}=\frac{c}{d}=\frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$

We know form (4) that

$$\frac{c}{a}=\frac{d}{b}$$

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}=\frac{d}{b}$$

Going back to the jam problem above. We got the simple equation

$$\frac{3}{2}=\frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x=27\cdot 2$$

$$x=\frac{27\cdot 2}{3}=\frac{3\cdot 9\cdot 2}{3}=18$$

1. In a department store, there is a sale of 25% off on everything. How much does the dress cost if its price before sale was $80? How much this dress will cost if an additional sale of 30% of will be applied?

2. There are 40000 books in a library. 75% of all books are in English, 10% of all books are in Spanish and the rest of the books are in French and German. How many books are there in the library in English and in Spanish?

3. Grapes were dried to raisin. During the process, the weight of grapes was reduced by 70%. How many kilograms of raisin was produced from 200 kg of grapes? How many kilograms of grapes were dried if the weight of obtained raisin is 15 kg?

4. In a dried fruit mix, there are 7parts of dried apples, 4 parts of dried pears and 5 parts of dried apricots. What is the weight (how many grams) of apples, pears, and apricots in the fruit mix, if the total weight of the mix is 1600g?

5. In order to prepare a homemade dried fruits and nuts mix Mary took 6 parts of raisins, 5 parts of dried cranberries and 3 parts of walnuts. Cranberries and walnuts altogether weighted 2 kg 400 g. What was the weight of the mix that Mary prepared?

6. To do her homework, Julia solved math problems, wrote an essay, and did a history project. It took her 2 hours and 15 minutes to finish all the assignments. The ratios of the times she spends doing math, writing the essay, and doing history project are 3:2:1. How much time did she spend for each of her subjects?

7. A book is 25% more expensive then a notebook. How many percent the notebook is less expensive than the book?

8. Dry cranberries contain 25% of water. How much water should be evaporated from 5 kg of fresh cranberries to get dry cranberries, if fresh cranberries contain 85% of water?

9. Three solutions of salt with concentration 10%, 15%, and 30% (it means that in the solution there are 10% (or 15%, or 30%) of the total mass is NaCl and 90% (or 85%, or 70%) is water) are mixed together. The mass of the first solution is 180g, mass of the second solution is twice as the mass of the first solution, and the mass of the third solution is 100 g. greater than the mass of the second solution. What is the concentration of the mixture?

10. Solve the following equations (hint: use the property of proportions):

$a.\ \frac{x}{7.2} = \frac{1\frac{1}{9}}{0.25};\qquad b.\ \frac{2\frac{1}{3}}{0.6x} = \frac{2.5}{1\frac{2}{7}};\qquad c.\ \frac{\frac{7}{12}}{0.14} = \frac{50x}{4.8};\qquad d.\ \frac{1\frac{3}{17}}{13.75} = \frac{2\frac{2}{11}}{3x}$

11. The ratio of boys to girls in 6th grade is $\frac{9}{11}$. The ratio of girls to boys in 7th grade is $\frac{31}{29}$. There are 100 and 120 students in 6th and 7th grades correspondingly, what is a ratio of boys to girls at the dance for 6 and 7 grade students, if all students came to the dance.

12. Dry apricots contain 22% of water. How much fresh apricot were used to produce a 200 g package of dry apricot if fresh apricots contain 85% of water?

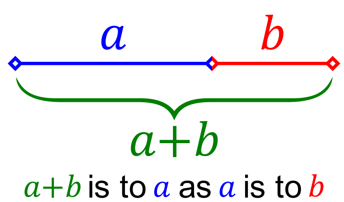
Famous ratios.

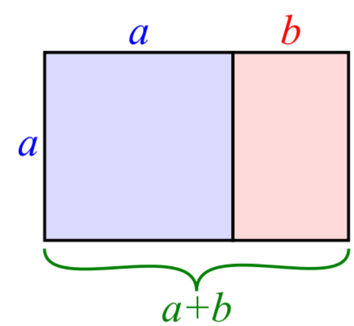
i. Let's measure the circumference and the diameter of a circle.

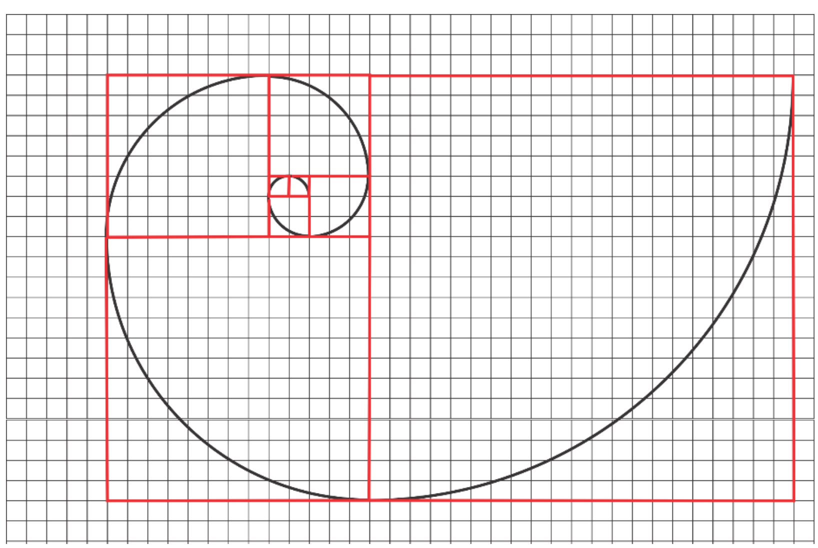
$$\frac{l}{d} = \pi$$

ii. Golden ratio:

$$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$$



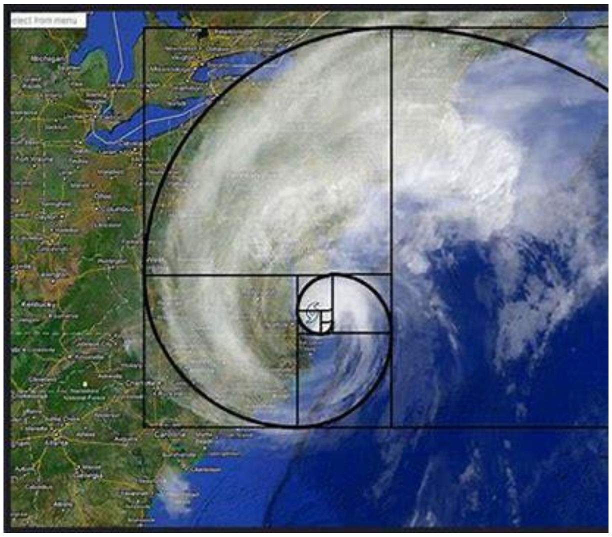


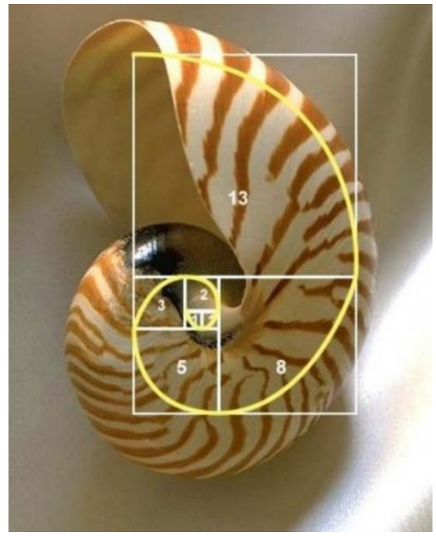


Fibonacci sequence:

$1, 1, 2, 3, 5, 8 \ldots..$

$F_n = F_{n-1} + F_{n-2}$

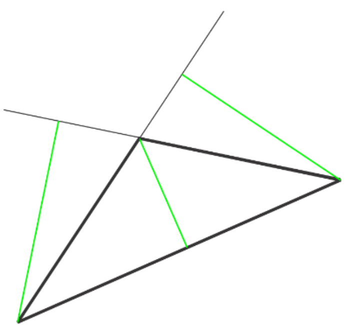


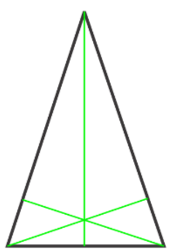


Geometry.

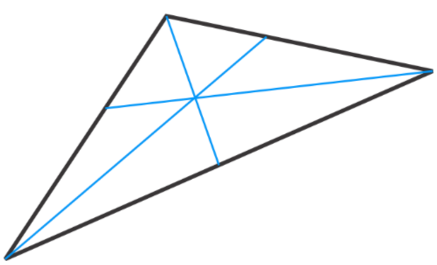
Special segments of a triangle.

From each vertex of a tringle to the opposite side 3 special segment can be constructed.





An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.



An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.

A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.