Class work 5.

Algebra.

Positive rational number is a number which can be represented as a ratio of two natural numbers:

$$a = \frac{p}{q}; \qquad p, q \in N$$

As we know such number is also called a fraction, p in this fraction is a nominator and q is a denominator. Any natural number can be represented as a fraction with denominator 1:

$$b = \frac{b}{1}; \;\; b \in N$$

Basic property of fraction: nominator and denominator of the fraction can be multiplied by any non-zero number n, resulting the same fraction:

$$a = \frac{p}{q} = \frac{p \cdot n}{q \cdot n}$$

In the case that numbers p and q do not have common prime factors, the fraction $\frac{p}{q}$ is irreducible fraction. If $p < q$, the fraction is called "proper fraction", if $p > q$, the fraction is called "improper fraction".

If the denominator of fraction is a power of 10, this fraction can be represented as a finite decimal, for example,

$\frac{37}{100} = \frac{37}{10^2} = 0.37,$ $\qquad \frac{3}{10} = \frac{3}{10^1} = 0.3,$ $\qquad \frac{12437}{1000} = \frac{12437}{10^3} = 12{,}437$

$$10^n = (2 \cdot 5)^n = 2^n \cdot 5^n$$

$$\frac{2}{5} = \frac{2}{5^1} = \frac{2 \cdot 2^1}{5^1 \cdot 2^1} = \frac{4}{10} = 0.4$$

$$\begin{array}{r|l} & 0.875 \\ \hline 8 & 7.000 \\ & -6.4 \\ & \;\;\;60 \\ & \;\;-56 \\ & \;\;\;\;\;40 \\ & \;\;\;-40 \\ & \;\;\;\;\;\;\;0 \end{array}$$

Therefore, any fraction, which denominator is represented by $2^n \cdot 5^m$ can be written in a form of finite decimal. This fact can be verified with the help of the long division, for example $\frac{7}{8}$ is a proper fraction, using the long division this fraction can be written as a decimal $\frac{7}{8} = 0.875$. Indeed,

$$\frac{7}{8} = \frac{7}{2 \cdot 2 \cdot 2} = \frac{7 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{7 \cdot 5^3}{2^3 5^3} = \frac{7 \cdot 125}{(2 \cdot 5)^3} = \frac{875}{10^3} = \frac{875}{1000} = 0.875$$

Also, any finite decimal can be represented as a fraction with denominator 10^n.

$0.375 = \frac{375}{1000} = \frac{3}{8} = \frac{3}{2^3};$ $\qquad 0.065 = \frac{65}{1000} = \frac{13 \cdot 5}{5^3 2^3} = \frac{13}{5^2 2^3};$

$6.72 = \frac{672}{100} = \frac{168}{25} = \frac{168}{5^2};$ $\qquad 0.034 = \frac{34}{1000} = \frac{17 \cdot 2}{5^3 2^3} = \frac{17}{5^3 2^2};$

```
  0.71428571...
7|5.000
-0 0
 5 0
-4 9
   10
  - 7
    30
  - 28
     20
   -14
      60
    -56
       40
     -35
        50
      -49
         10
        .....
```

In other words, if the finite decimal can be represented as an irreducible fraction, the denominator of this fraction will not have other factors besides 5^m and 2^n. Converse statement is also true: if the irreducible fraction has denominator which only contains 5^m and 2^n than the fraction can be written as a finite decimal. (Irreducible fraction can be represented as a finite decimal if and only if it has denominator containing only 5^m and 2^n as factors.)

If the denominator of the irreducible fraction has a factor different from 2 and 5, the fraction cannot be represented as a finite decimal. If we try to use the long division process, we will get an infinite periodic decimal.

At each step during this division we will have a remainder. At some point during the process we will see the remainder which occurred before. Process will start to repeat itself. On the example on the left, $\frac{5}{7}$, after 7, 1, 4, 2, 8, 5, remainder 7 appeared again, the fraction $\frac{5}{7}$ can be represented only as an infinite periodic decimal and should be written as $\frac{5}{7} = 0.\overline{714285}$. (Sometimes you cen find the periodic infinite decimal written as $0.\overline{714285} = 0.(714285)$).

How we can represent the periodic decimal as a fraction?

Let's take a look on a few examples: $0.\bar{8}$, $2.35\bar{7}$, $0.\overline{0108}$.

1. $0.\bar{8}$. $x = 0.\bar{8}$ $10x = 8.\bar{8}$ $10x - x = 8.\bar{8} - 0.\bar{8} = 8$ $9x = 8$ $x = \frac{8}{9}$	2. $2.35\bar{7}$ $x = 2.35\bar{7}$ $100x = 235.\bar{7}$ $1000x = 2357.\bar{7}$ $1000x - 100x = 2357.\bar{7} - 235.\bar{7}$ $= 2122$ $x = \frac{2122}{900} = \frac{1061}{450}$	3. $0.\overline{0108}$ $x = 0.\overline{0108}$ $10000x = 108.\overline{0108}$ $10000x - x = 108$ $x = \frac{108}{9999} = \frac{12}{1111}$

Now consider $2.4\bar{0}$ and $2.3\bar{9}$

$x = 2.4\bar{0}$
$10x = 24.\bar{0}$
$100x = 240.\bar{0}$

$100x - 10x = 240 - 24$
$x = \frac{240 - 24}{90} = \frac{216}{90} = 2.4$

$x = 2.3\bar{9}$
$10x = 23.\bar{9}$
$100x = 239.\bar{9}$

$100x - 10x = 239 - 23$
$x = \frac{239 - 23}{90} = \frac{216}{90} = 2.4$

Exercises.

1. Represent the following fractions as decimals:

$a.\ \frac{3}{2000},$ $b.\ \frac{17}{40};$ $c.\ \frac{28}{140};$

$d.\ \frac{7}{4};$ $e.\ \frac{3}{2};$ $f.\ \frac{9}{5};$

$g.\ \frac{123}{20};$ $h.\ \frac{783}{540};$ $i.\ \frac{324}{25};$

2. Write as a fraction:

$a.\ 0.\overline{3},$ $b.\ 0.3,$ $c.\ 0.\overline{7},$ $d.\ 0.7,$

$e.\ 0.1\overline{2},$ $f.\ 0.\overline{12},$ $g.\ 0.12,$ $h.\ 1.12\overline{3},$

$i.\ 7.5\overline{4},$ $j.\ 1.0\overline{12}.$

3. Evaluate the following using decimals:

$a.\ 0.36+\frac{1}{2};\quad b.\ 5.8-\frac{3}{4};\quad c.\ \frac{2}{5}:0.001;\quad d.\ 7.2\cdot\frac{1}{1000}$

4. Evaluate the following using fractions:

$a.\ \frac{2}{3}+0.6;\quad b.\ 1\frac{1}{6}-0.5;\quad c.\ 0.3\cdot\frac{5}{9};\quad d.\ \frac{8}{11}:0.4;$

5. Evaluate:

$a.\ \frac{5\frac{1}{7}}{3\frac{3}{14}};\quad b.\ \frac{1\frac{1}{3}\cdot 2\frac{3}{11}\cdot 3\frac{1}{2}}{\frac{1}{2}\cdot 4\frac{1}{6}\cdot 3\frac{9}{11}};\quad s.\ \frac{1\frac{1}{2}\cdot 2\frac{2}{3}\cdot 0.36}{0.6\cdot 2\frac{1}{4}\cdot 1\frac{1}{3}};\quad d.\ \frac{0.38\cdot 0.17\cdot 2\frac{2}{15}\cdot 2.7}{5.1\cdot 3\frac{4}{5}\cdot 0.064}$

6. Compare:

Example: What is greater 31^{11} or 17^{14}?

We can see that $31 < 32 = 2^5;\ \ 2^4 = 16 < 17,$

$$31^{11} < 32^{11} = (2^5)^{11} = 2^{55}$$

$$(17)^{14} > 16^{14} = (2^4)^{14} = 2^{56}$$

We can write the following:

$$31^{11} < 32^{11} = 2^{55} < 2^{56} = 16^{14} < (17)^{14}$$

$$31^{11} < 17^{14}$$

a. 127^{23} and 513^{18}

b. 9997^{10} and 100003^{8}

c. 5^{300} and 3^{500}

7. Using distributive property of addition and multiplication factorize the following expressions.

Example:

Example:

$2a + ax = a \cdot (2 + x)$

$a.$ $3x + 6;$

$b.$ $cb - y^2c$

$c.$ $x^3 - 5x^2;$

$d.$ $yx + x^2;$

8. Write the following numbers as the power of base 10:
    a. 10, 100, 1000, 10000, 100000, 1000000
    b. 0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001

9. Write in the in ascending order
    a. $-1.2, -1.2^2, 1.2, (-1.2)^2$
    b. $0.15, -0.15, (-0.15)^2, (-0.15)^3$

10. Simplify the expressions:

$aa^m(-a)^2;$

$c^kc(-c^2)c^{k-1}c^3;$

$d^nd(-d^{n+1})d^nd^2;$

$2x^2y^3 \cdot (-4xy^2);$

$2^4 + 2^4;$

$2^m + 2^m;$

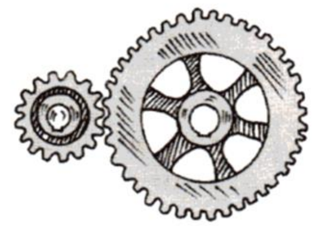
$2^m \cdot 2^m;$

$3^2 + 3^2 + 3^2;$

$3^k + 3^k + 3^k;$

$3^k \cdot 3^k \cdot 3^k;$

11. Two gears are in in clutch. One gear has 18 cogs, and another has 63. How many turns will each gear make before they both return to their original position?

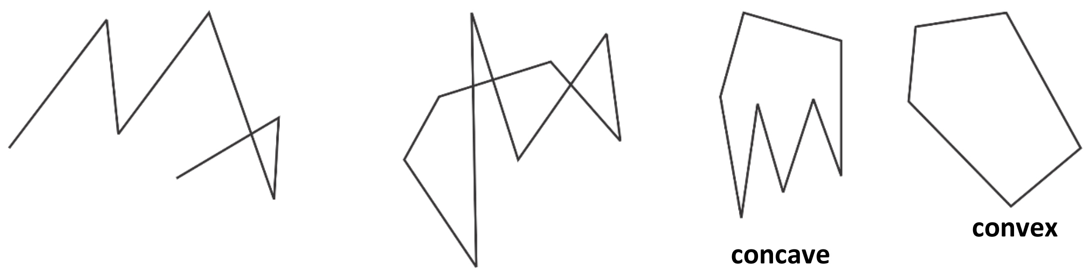


Geometry.

Polygons.

Draw a chain of segments, so that the last point of one segment is a first point of the next, and two consecutive points don't lie on the same line.

Draw such chain so that the last point of the last segment is the first point of the first one. We got a closed broken line. Is this a sufficient condition to get a polygon?

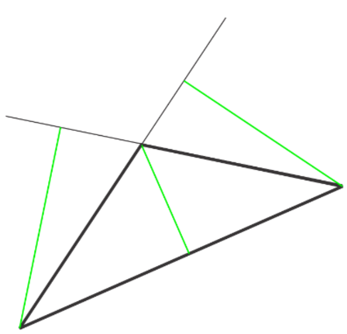


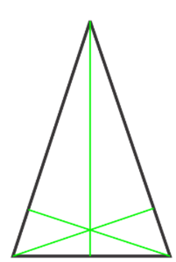
In geometry, a **polygon** is a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain. These segments are called its *edges* or *sides*, and the points where two edges meet are the polygon's *vertices* (singular: vertex) or *corners*. The interior of the polygon is sometimes called its *body*. An ***n*-gon** is a polygon with *n* sides; for example, a triangle is a 3-gon.

- *What is the difference between convex and concave polygons?*

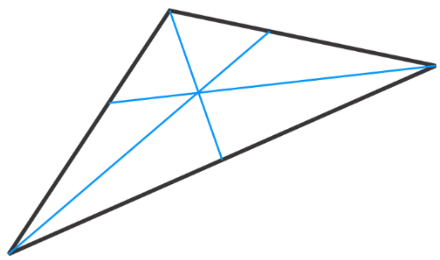
The simplest polygon is a triangle.

Draw a triangle. Measure its angles. Add them. How much did you get?



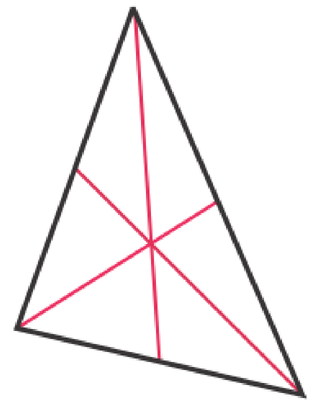


An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.



An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.

A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.



12. Draw the isosceles
    a. right triangle
    b. acute triangle
    c. obtuse triangle

13. Draw a triangle with sides 3 cm, 5 cm, and the angle between them 50°.

14. Draw a triangle with angles 30° and 50° and the side between them 7 cm. Do we need another information to construct a triangle?

15. Draw three triangles, in one triangle draw three medians, in the second triangle draw three bisectors, in the third triangle draw three altitudes.