Classwork 3.

Algebra.

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5$,

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

fundamental theorem of arithmetic:

Any natural number greater than 1 either is a prime number or can be represented as a product of prime numbers and such representation is unique.

For example:

$$1200 = 5 \cdot 2 \cdot 5 \cdot 2 \cdot 3 \cdot 2 \cdot 2$$

The theorem says two things, for this example: first, that 1200 can be represented as a product of primes, and second, that no matter how this is done, there will always be exactly four 2s, one 3, two 5s, and no other primes in the product. This theorem is a main reason why 1 is not considered a prim number.

LCM and GCF (GCD).

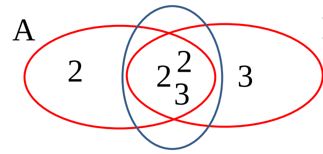
Each natural number is a prime number or can be represented as product of a unique set of prime numbers (see above, fundamental theorem of arithmetic). How we can find all divisors of a number? If the number is prime, there is no other divisors, but itself. If the number is not prime – each prime factor is a divisor, as well as product of all possible combinations (subsets of the set of prime factors). For example, number 24 has a prime representation $24 = 2 \cdot 2 \cdot 2 \cdot 3$, therefore 4, 8, 6, and 12, as well as 24, will be also the divisors of 24. Any two (or more) natural numbers can have common divisors (such that both numbers can be divided by evenly), or, in case that there are no such common divisors, they are called mutually prime. For example, 9 and 20:

$$9 = 3 \cdot 3, \qquad 20 = 2 \cdot 2 \cdot 5$$

These two numbers are not prime, but because they don't have common divisors, they are mutually prime.
How to find common factor? Take a look at the prime factorization of numbers 24 and 36:

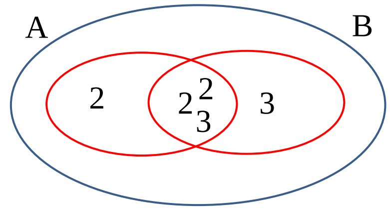
$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

Both numbers have common factors, so they both can be divided two times by 2 and by 3, and by the product of any combinations of these three numbers. The greatest divisor (greatest common factor) will be the product of all common factors. This can be also be represented as the Venn diagrams of the sets of prime factors of numbers 24, 36, and the intersection of these two sets. Set $\mathrm{A} = (P, 24)$, $\mathrm{B} = (P, 36)$



Multiple of a number a is any number, which is divisible by a. $(M = na,\ n \in N)$. For two numbers, a and b common multiple is a number which is divisible by both, a and b. One of the common multiples is the product of a and b, but it's not necessarily the smallest one.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$



The product of the common prime factors together with the remaining prime factors from both numbers will be divisible by both numbers, and will be the smallest multiple. In terms of set theory, this will be the product of all elements of the unity of sets A an B.

Exercises:

1. Find all prime factors of the following numbers:

   66, 28, 128, 555, 1233

   Example: $66 = 2 \cdot 3 \cdot 11$

2. Do the prime factorization of the numbers:

   34, 40, 100, 225, 1000

3. Find GCD (GCF) of

   420 and 450,

   810, 945 and 1125

4. Find LCM of

   a. 8 and 12

   b. 15, 18, and 21

5. Dunno boasted ability to multiply in the mind. To test it, Doono suggested writing some number, multiplying its digits, and saying the result. “2178,” Dunno immediately blurted out, only having had time to write down the number. "It cannot be," - replied, thinking, Doono. How did he detect the error without knowing the source number?

6. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12 minutes to complete the round-trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?

7. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

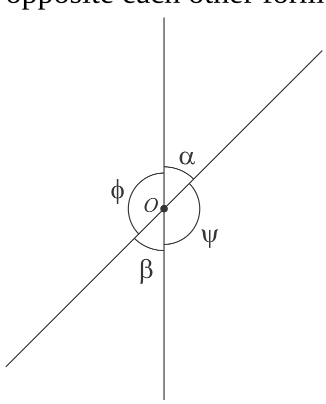
8. Find the GCF and LCM for the following prime factorized numbers:
   $a.$ $a = 2 \cdot 3 \cdot 5,$ $b = 2 \cdot 3 \cdot 11$
   $b.$ $a = 3 \cdot 3 \cdot 7 \cdot 7,$ $b = 2 \cdot 2 \cdot 2 \cdot 5$
   $c.$ $a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 7,$ $b = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

9. Is the number a is divisible by number b?
   $a.$ $a = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 11,$ $b = 2 \cdot 2 \cdot 11$
   $b.$ $a = 3 \cdot 3 \cdot 5 \cdot 13,$ $b = 2 \cdot 13$
   $c.$ $a = 2 \cdot 3 \cdot 5 \cdot 5 \cdot 17,$ $b = 2 \cdot 3 \cdot 3 \cdot 5$

10. “Sweet Mathematics” chocolate candies are sold in 12 pieces per box, and “Geometry with Nuts” are sold by 15 pieces per box. What is the smallest number of boxes of both types of chocolates you need to buy so that the two chocolates are equally divided?

Geometry.

When two straight lines intersect at a point, four not straight angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem:

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

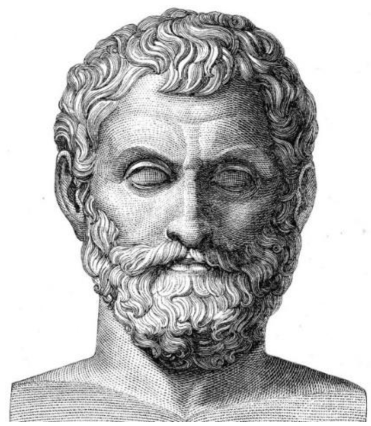
Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$, therefore, we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Exercises.

11. Draw 3 different angles, measure them with a protractor.
12. Draw angles with the measures $72°,\ 155°, 90°$. Use ruler and protractor.
13. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
14. Three straight lines intersect at one point. How many non straight angles a re formed?
15. Do the operations with angular measures:

$a.\ 25°36'24'' + 36°24'40''$ $b.\ 48°26' + 28°36'34''$

$c.\ 48°48'^{48''} - 24°36'36''$ $d.\ 3 \cdot 24°36'$

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