Homework 16

Tides

We recently discussed probably the most apparent effect of the moon's gravity which can be observed on Earth – tides. Almost everyone knows that the reason for the tides is the moon's gravity, but much fewer people can explain why the tide period is 12 hours but not 24 hours. A typical (**incorrect!**) explanation is that the water of the oceans is pulled towards the moon so the water surface is "bulged" at the side of the Earth that faces the moon. As long as the Earth rotates, the bulge periodically changes its position with respect to the continents and we have high and low tides. This seemingly logical explanation is in contradiction with the 12 hour period of the tides – as long as there is just one bulge and the period of the Earth rotation is 24 hours we could expect same period for the tides.

Let us try to give a more consistent explanation. First, because the tide period is 12 hours, there should be 2 water bulges at opposite sides of Earth. One is at the side which is closest to the moon, the other one – at the opposite side. First, we have to ask: why is the water bulged in a particular place? Probably because the water weight is less in this place. As long as there are 2 bulges at the axis connecting the centers of the Earth and the Moon (see Figure 1 below), the Moon's gravity somehow reduces the weight of the water in these places.

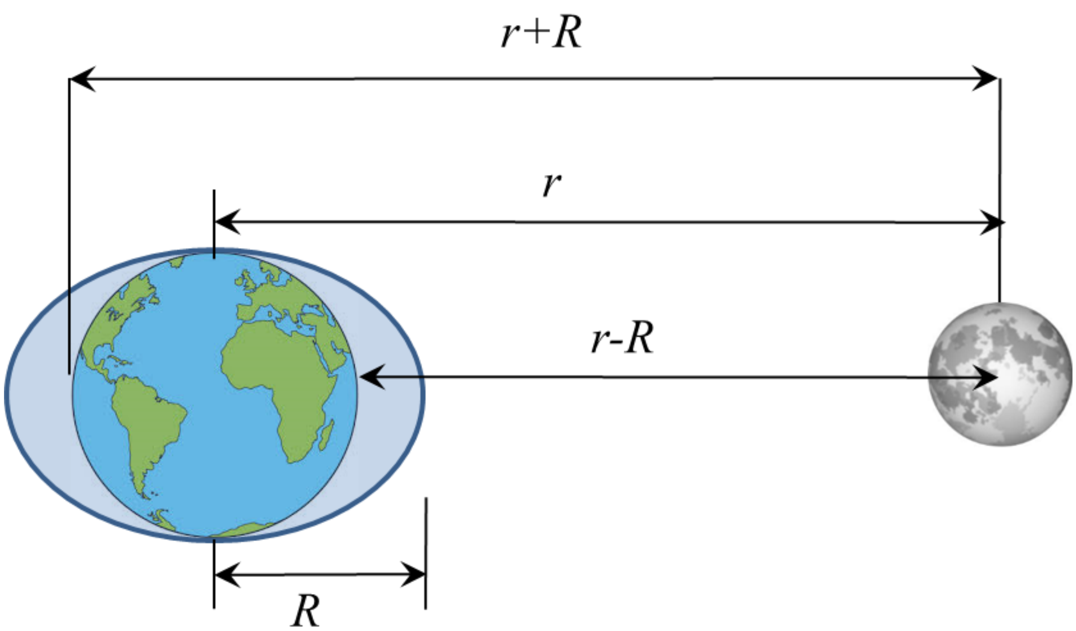

Figure 1. Here R is the radius of the Earth, r is the distance between the centers of the Earth and the Moon.

The weight of an object is determined by acceleration of the object with respect to Earth's surface. Imagine that the Earth and all the objects on it including us are independently subjected to the same acceleration. Will we feel the change in our weight? I would say no. For example, the objects in a free falling elevator are weightless since both the object and the elevator are equally accelerated by Earth. The change in the object's weight caused by the Moon is determined by the "***difference***" between the accelerations of Earth and the object. As long as the Moon would have provided the same acceleration to Earth and all the objects on it, there would be no tides. However, this is not the case. As long as the gravity force depends on the distance between objects, additional accelerations given by the Moon to the Earth and to the objects on the Earth's surface are different. To calculate correction to the object's weight due to these additional accelerations we have to subtract the additional acceleration of the Earth due to the Moon's gravity from the additional acceleration of an object on the Earth surface. Before doing that I would like to explain why I am using accelerations instead of forces. Gravity force is proportional to an object's mass and is different for different objects. Acceleration due to gravity is the same for all the objects since it is depends only on the planet's mass and the position of the object.

The magnitude of acceleration of the Earth due to the Moon's gravity is:

$$a_E = \frac{F_{gravity}}{M} = G\frac{m}{r^2} \qquad (1)$$

Here m and M are masses of the Moon and the Earth, G is the gravitational constant. Acceleration of an object on the Earth surface which faces the Moon is:

$$a_c = G\frac{m}{(r-R)^2} \qquad (2)$$

As we can see, objects on the Earth surface which faces the Moon experience higher acceleration than the Earth since the object is closer to the Moon and the gravity force increases with the distance. Acceleration of an object on the Earth surface which is opposite to the Moon is:

$$a_f = G\frac{m}{(r+R)^2} \qquad (3)$$

The correction to the object's weight is determined by the difference of the object's and the Earth's accelerations. On the Earth side which faces the Moon it is:

$$\Delta a_c = a_c - a_E = G\frac{m}{(r-R)^2} - G\frac{m}{r^2} = \frac{GmR(2r-R)}{r^2(r-R)^2} \qquad (4)$$

Since R<<r, we can neglect R in the parenthesis in both numerator and denominator. So we obtain:

$$\Delta a_c \approx \frac{2GmR}{r^3} \qquad (5)$$

Similarly, for the object on the Earth's surface which is opposite to the Moon we obtain:

$$\Delta a_f \approx -\frac{2GmR}{r^3} \qquad (6)$$

The "minus' sign means that in this case the additional acceleration is directed oppositely to the additional acceleration on the "facing the Moon" side. The situation is shown in Figure 2.

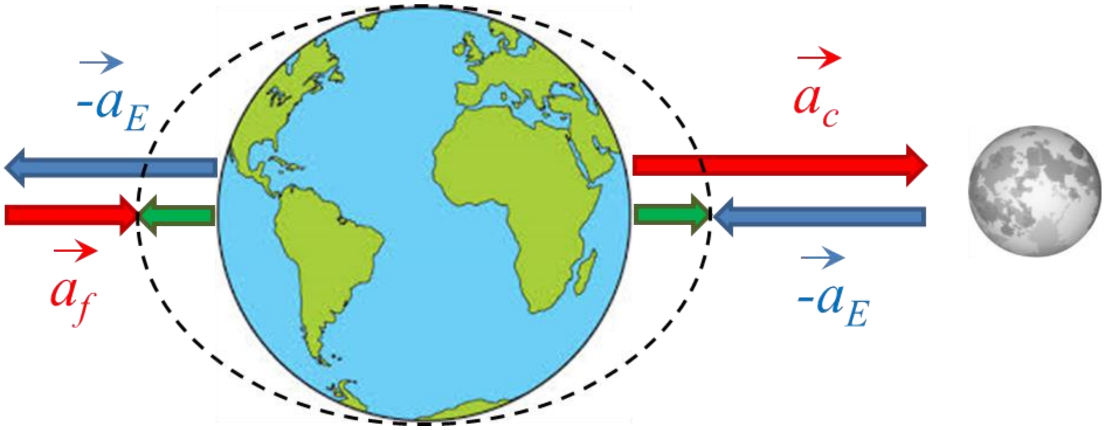

Figure 2. $\Delta \boldsymbol{a}_c$ and $\Delta \boldsymbol{a}_f$ are represented by the green arrows.

What happens to the object's weight at the "halfway"? As it is shown in Figure 3, in this case the accelerations of the Earth and the object are not parallel.

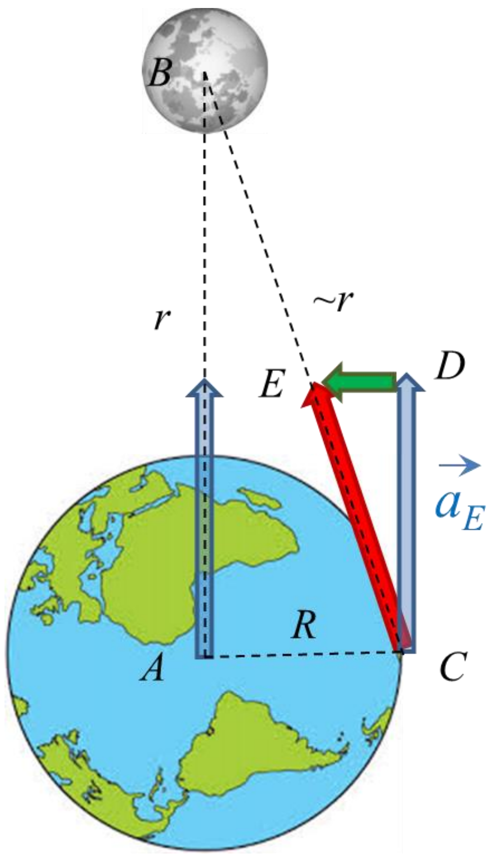

Figure 3. Additional acceleration in point C is shown as the green arrow.

In this case, the additional acceleration is directed towards the Earth center, so the weight of the object will increase. The magnitude of the additional acceleration *Δa* in point C can be found from similarity of triangles ABC and CDE:

$$\frac{\Delta a}{a_E} = \frac{R}{r} \Rightarrow \Delta a = \frac{GmR}{r^3} \qquad (7).$$

These additional accelerations are called ***tidal*** accelerations and corresponding forces – ***tidal*** forces. If we calculate tidal acceleration using equation (5) we obtain 10^{-6}m/s^2, which is one ten millionths part of ***g***. This correction is small. However, it increases the water level at the Earths side which faces the Moon by ~54cm.

Tidal waves produce friction force which slows down the Earth's rotation. There is the hypothesis that strong tidal friction stopped the Moon's rotation when the Moon was in liquid state long time ago. So now we can see just one side of the Moon.

Problem:

1. Calculate magnitude tidal acceleration on the Earth due to the gravitational attraction of the Sun.
2. The length of the day is increased due to tidal friction by 2.3 millisecond each century. Estimate tidal friction force applied to the Earth surface. Consider the Earth as an ideal sphere with radius of 6,400 km and mass 6x10^{24}kg. Just to remind: the moment of inertia of a ball is 5mR^2/2.