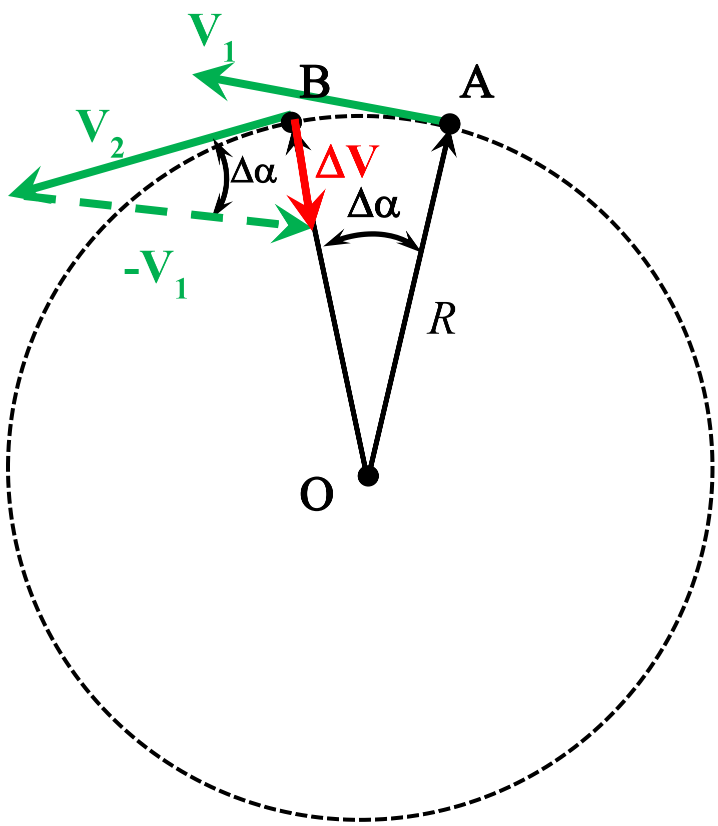
Homework 2

Centripetal acceleration.

We already know that in uniform circular motion, the acceleration vector is inward. How to calculate the magnitude of the centripetal acceleration?



Imagine that an object is moving along a circle from point A to point B (see the picture above). The speed of the object is the same in both points (since it is *uniform* circular motion). The velocity vectors are different. As the object travels from point 1 to point 2, its velocity vector turns. We know that at the circular motion the velocity at any point is directed along the tangent line to this point. The tangent line is perpendicular to the radius drawn from the center to the "tangent" point. Simply speaking, each black arrow in the figure above is perpendicular to the corresponding green arrow. So, the angle $\Delta\alpha$ between the velocities V_1 and V_2 is equal to the angle $\Delta\alpha$, swept by the moving object.

As we remember, acceleration is the change in velocity divided by the time, required for this change. Change in velocity ΔV is shown by the red arrow in the picture above. To find it we have to subtract vector ***V1*** from the vector ***V2***. To do that we will prepare the vector ***–V1*** which has the same length as ***V1,*** but its direction is opposite. Then we add ***–V1*** to ***V2***.

We have to find the length of the red arrow ΔV and divide it by the time $\boldsymbol{\Delta t}$ which is required for the object to travel from point A to point B.

$$a = \frac{\Delta V}{\Delta t} \quad (1)$$

To calculate ΔV we assume that the arc AB is really really small, so the arc AB is very close to a straight line. We can see that

$$|AB| \approx R \cdot \Delta\alpha \quad (2)$$

It follows from our way to measure the angle. It turns that the formula above is good for any "narrow" isosceles triangle! To find a "short" side we have to multiply one of the "long" sides to the small angle between them. The "narrower" the triangle, the more exact is the formula (2). Let us apply this formula to the triangle formed by V_1, V_2 and ΔV:

$$\Delta V \approx V \cdot \Delta\alpha \qquad (3)$$

Let us plug ΔV from the formula (3) to the formula (1):

$$a = \frac{\Delta V}{\Delta t} \approx \frac{V \cdot \Delta\alpha}{\Delta t} \qquad (4)$$

But $\Delta\alpha/\Delta t$ is the angular velocity $\boldsymbol{\omega}$. So we can write:

$$a = V \cdot \omega \quad (5)$$

If we remember that $\boldsymbol{\omega = V/R}$, the magnitude of the acceleration can be written as

$$a = \frac{V^2}{R} \quad (6),$$

where *R* is the radius of the circle. So we can see that the faster we move and the sharper we turn the more acceleration we experience and the more centripetal force we need to complete the turn.
Now, as long as we know that an object moves along a circle with a radius R with a constant speed V, we can immediately write its acceleration using the formula (6)!

Problems:

1. The weight of a pebble on the North (or South) pole is 100g. What is the weight of this pebble near the equator? *Assume that the Earth is a perfect sphere.*

2. A car is moving through a semicircle bridge with a radius of 30m at a speed of 10m/s. What is the weight of the car in the upper point of the bridge?