Homework 22.

Equation of state of the ideal gas.

Unlike simple mechanical systems, gases can not be practically characterized by coordinates and velocities of each molecule – there is just not possible to perform calculations over billions of billions of molecules. Instead we can pick up some average parameters such as temperature T, pressure P and volume V (volume here is the volume of jar or bottle where the gas is kept). Pressure is measured in N/m^2 (Pa), Volume in m^3, Temperature in degrees according to Kelvin's scale (K). One Kelvin's degree equals to 1 Celsius degree, but zero at the Celsius scale is 273 degrees at the Kelvin's scale (or simply 273K). For example, room temperature is 27^oC, but 300K.

For ideal gas (we call gas "ideal" if the molecules do not interact with each other -they do not attract or repulse) T,P and V are connected by a simple equation:

$$PV = nRT,$$

where *n* is the number of molecules measured in *moles*, R is a constant which equals 8.31 J/(mole K), or 8.31×10^3 J/(kmole K). One mole is a certain *number* of atoms or molecules. If we take $\sim 6 \times 10^{23}$ molecules of, say, water we will have one mole of water (6.02×10^{23} is called Avogadro's number). 1 kilomole (kmole) = 1000 moles.

The number is named after Amedeo Avogadro, italian physicist:

Amedeo Avogadro (1776-1856)

How to find the number of moles (or kmoles) if we know a mass of a substance? First we have to find a mass of one molecule of the substance. To do that, we need to take a look into the periodic table of elements.

Periodic table of elements

Chemical elements are the "building blocks" of nature. All the objects around us are "constructed" from chemical elements. Despite great variety of the objects and substances around us there are only 118 chemical elements (some of them are not shown in the table below). They are systematized and arranged in the table which is called periodic table of elements.

hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 * *	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
* *Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

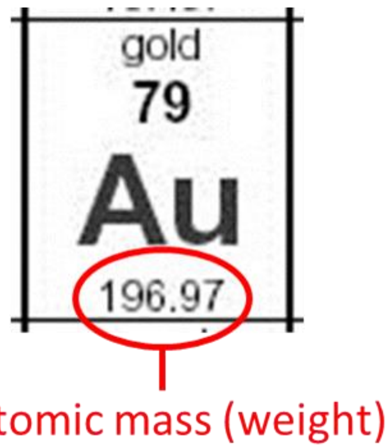
Periodic table of elements.

The periodic table was first suggested by a Russian chemist Dmitri Mendeleev in 1869. He found that if the chemical elements are arranged according their atomic mass, their chemical properties exhibit periodicity, that is why it is called "periodic".

Dmitri Mendeleev (1834-1907).

Only two of the chemical elements – mercury and bromine - are liquids at normal conditions (T=300K, atmospheric pressure), eleven elements are gases. The other elements are solids except nine elements (109-111 and 113-118) in the end of the table whose chemical properties are still unknown.

At the bottom of each cell of the table there is a number which represent the mass of the atom (atomic mass) in the so cold atomic units of mass.



One atomic unit of mass is $1.66x10^{-24}$g, or $1.66x10^{-27}$kg. It was chosen in such a way that if we take 1 mole of a substance (that means $6.02x10^{23}$ molecules), the mass of this 1 mole will be numerically equal to the atomic mass, but in grams.

For example, atomic mass of hydrogen (H) is ~1. A molecule of hydrogen consists of 2 atoms, so the mass of the molecule is 2 atomic units of mass. If we take $6.02x10^{23}$ molecules of hydrogen (1 mole), the total mass of the gas is 2g, or 0.002kg.

Another example: how many molecules (or moles) in 100g of water? A molecule of water consists of two atoms of hydrogen and one atom of oxygen. Let's look in the periodic table. Atomic mass of hydrogen is 1, atomic mass of oxygen is 16. So the mass of a molecule of water expressed in atomic units is 18. It means that a mass of 1 mole of water is 18g (we can say that *molecular mass* of water is 18g). Now we can easily find how many moles in 100g of water: 100g:18g/mole~5.56moles. And we have total $5.56x6.02x10^{23}=33.44x10^{23}=3.34x10^{24}$.

Now, the problems (Please do not forget – we must use Kelvin's scale for temperature):

1. There is a 1 liter bottle filled with water at 27^oC. The water is liquid at this temperature because there is attracting force between the molecules. Imagine, that we have suddenly "turned off" this attracting force. What is the pressure in the bottle now?
2. What is the volume of 1 mole of an ideal gas at the temperature of 27^oC and pressure 105,000 N/m^2?
3. This problem is a bit more challenging: find the formula which express the density of an ideal gas through its molecular mass ($T=27^oC$, $P=105,000\ N/m^2$).