MATH 9: REVIEW 3

2021/05/02

1. Complex Numbers

The basics of complex numbers:

- The set $S = \{z \in \mathbb{C} | z\overline{z} = 1\}$ is closed under multiplication. This means that if z_1 and z_2 are in S, then so is $z_1 \cdot z_2$. Any nonzero complex number can be written as a product of an element in S and an element in $\mathbb{R}^+$. This is called the polar form. ($\mathbb{R}^+ = \{x \in \mathbb{R} | x > 0\}$)
- Or, using the coordinate geometry of real and imaginary parts, we can define the angle of a complex number z as the angle between the ray from 0 to 1 with the ray from 0 to z. If the angle is ϕ, then a complex number $z = a + bi$ can be written as $z = r(\cos(\phi) + i\sin(\phi))$. This is called the polar form. (r is some positive real number.)
- Laws concerning the way that complex numbers are added or multiplied can be used to derive other useful formulas or concepts. For example, multiplication of complex numbers in polar form can be used to prove the cosine and sine angle sum formulas.
- De Moivre's Formula: $(\cos\phi + i\sin\phi)^n = \cos(n\phi) + i\sin(n\phi)$

2. Scavenger Hunt!

1. State the 3 properties of the center of mass for a system of point masses.

2. Prove the rational root theorem.

3. Prove the AM-GM inequality.

4. State the principle of mathematical induction. Explain the following terms: base case, inductive step, inductive hypothesis. Explain the difference between a normal inductive hypothesis and a strong inductive hypothesis.

5. Explain the duality of the algebra of sets.

6. State the definition of a relation on sets. List a few different properties that a relation could have.

7. Prove Thales's Intercept Theorem: that if you have points B on ray $\overrightarrow{OB}$ and D on ray $\overrightarrow{OC}$ such that $\overleftrightarrow{AC} \| \overleftrightarrow{BD}$, then $\frac{OA}{OB} = \frac{OC}{OD}$

8. State and prove the property of the bisector that can be deduced from theorems about similar triangles.

9. State De Moivre's Formula.