MATH 9: REVIEW 1

2021/04/18

1. Problems

1. Perform division with remainder of x by $x+1$. That is, find a polynomials q, r whose degree is less than the degree of $x+1$ such that $x = q(x)(x+1) + r(x)$.
Reminder: polynomials can have any real numbers as coefficients, including negative numbers.

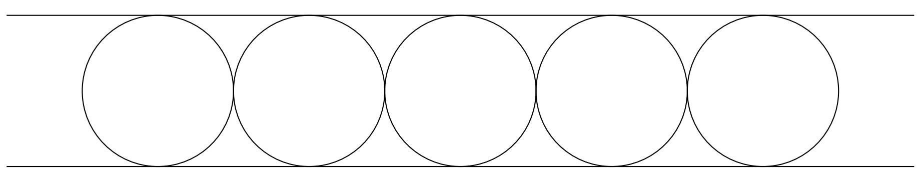
2. (a) Find the remainder of x^2 divided by x^2+1.
(b) Find the remainder of $(\sqrt{2}x)^2$ divided by x^2+1.
(c) Determine the remainder of $(cx)^2$ divided by x^2+1. Your answer should be in terms of c.
(d) Given some real number $r < 0$, find a degree-1 polynomial $p(x)$ such that $p^2(x)$ divided by x^2+1 has remainder r.

3. Suppose I have four real numbers a, b, r, s. I know the sum $a+b$, and I know the product rs. Suppose $a+b=5$, $rs=20$, and a, b, r, s are roots of $x^4 - 14x^3 + kx^2 - 154x + 120$. Determine the value of k.

4. Given some integer $t > 1$ and two distinct polynomials $p(x)$ and $q(x)$, prove that if the coefficients of p and q are all positive integers less than t, then $p(t) \neq q(t)$.

5. Write down the equation of a hyperbola whose axes are the x-axis and y-axis, and that goes through the points $(1, 1)$ and $(-1, -1)$.

6. Given two parallel lines m, l, consider a chain of circles between these lines that are all tangent to the two lines and to successive circles in the chain. Below is a figure displaying part of the chain. Now consider the inversion of every circle around every other circle in the chain (the resulting collection of inverted figures will have infinitely many figures). Draw out what the resulting collection of inverted figures would look like.



7. Let $\mathbb{Z}[x]$ be the set of all polynomials with integer coefficients. Is $\mathbb{Z}[x]$ a countable set?

8. Let $\mathbb{A}$ be the set of all real numbers that are roots of polynomials with integer coefficients. Is $\mathbb{A}$ a countable set? [The set $\mathbb{A}$ is known as the *algebraic numbers*.]