MATH 9: POLYNOMIALS

2021/03/07

1. Homework: Geometry

1. When solving inversion problems, the goal usually is to turn as many circles into lines as possible. This means that the center of inversion is the most important detail, and then the radius can thence be chosen as desired. Typically, you pick a center of inversion such that the most useful circles go through that point. Remember that "go through" simply means that the point is on the circle.
Suppose you have two externally tangent circles, with radii 1 and 2 ("externally tangent" means the smaller circle is not inside the bigger circle). Prove that inversion around the point of tangency sends the circles to parallel lines. Then calculate the distance between the resulting lines as a function of the radius of the circle of inversion.

2. Suppose you have a rectangle $ABCD$ and a point O inside the rectangle. Prove that inversion around O sends the rectangle to a cyclic quadrilateral $A'B'C'D'$.

3. Let λ be a circle with center K. Let A be some point on λ.

(a) Prove that inversion around A sends λ and $\overleftrightarrow{AK}$ to perpendicular lines.

(b) Prove that λ' is the perpendicular bisector of $\overline{A'K}$.

4. Let A and B be any two points.

(a) What is the locus of points B' of reflections of B around lines through A?

(b) Let m be the line through A perpendicular to $\overline{AB}$. What is the locus of points B' of inversions of B through circles with centers on m that go through A? [Hint: if O is on m, then $\angle OAB$ is a right angle. Where does this right angle end up after the inversion?]