February 28, 2021 Math 9

Algebra.

Polynomials and factorization.

Polynomial long division algorithm for dividing a polynomial by another polynomial of the same or lower degree, is a generalized version of the familiar arithmetic technique called long division. It can be done easily by hand, because it separates an otherwise complex division problem into smaller ones.

Example

Find $\frac{x^3 - 12x^2 - 42}{x - 3}$.

The problem is written like this:

$$\frac{x^3 - 12x^2 + 0x - 42}{x - 3}.$$

The quotient and remainder can then be determined as follows:

1. Divide the first term of the numerator by the highest term of the denominator (meaning the one with the highest power of x, which in this case is x). Place the result above the bar ($x^3 \div x = x^2$).

$$\begin{array}{r} x^2 \\ x - 3\overline{)x^3 - 12x^2 + 0x - 42} \end{array}$$

2. Multiply the denominator by the result just obtained (the first term of the eventual quotient). Write the result under the first two terms of the numerator ($x^2 \cdot (x - 3) = x^3 - 3x^2$).

$$\begin{array}{l} x^2 \\ x - 3\overline{)x^3 - 12x^2 + 0x - 42} \\ x^3 - 3x^2 \end{array}$$

3. Subtract the product just obtained from the appropriate terms of the original numerator (being careful that subtracting something having a minus sign is equivalent to adding something having a plus sign), and write the result underneath ($(x^3 - 12x^2) - (x^3 - 3x^2) = -12x^2 + 3x^2 = -9x^2$) Then, "bring down" the next term from the numerator.

$$
\begin{array}{rl}
 & x^2 \\
x - 3 & \overline{)x^3 - 12x^2 + 0x - 42} \\
 & \underline{x^3 - 3x^2} \\
 & \qquad -9x^2 + 0x
\end{array}
$$

4. Repeat the previous three steps, except this time use the two terms that have just been written as the numerator.

$$
\begin{array}{rl}
 & x^2 - 9x \\
x - 3 & \overline{)x^3 - 12x^2 + 0x - 42} \\
 & \underline{x^3 - 3x^2} \\
 & \qquad -9x^2 + 0x \\
 & \qquad \underline{-9x^2 + 27x} \\
 & \qquad\qquad\quad -27x - 42
\end{array}
$$

5. Repeat step 4. This time, there is nothing to "pull down".

$$
\begin{array}{rl}
 & x^2 \;\, - 9x \quad - 27 \\
x - 3 & \overline{)x^3 - 12x^2 + 0x - 42} \\
 & \underline{x^3 - 3x^2} \\
 & \qquad -9x^2 + 0x \\
 & \qquad \underline{-9x^2 + 27x} \\
 & \qquad\qquad\quad -27x - 42 \\
 & \qquad\qquad\quad \underline{-27x + 81} \\
 & \qquad\qquad\qquad\qquad -123
\end{array}
$$

6. The polynomial above the bar is the quotient, and the number left over (-123) is the remainder.

$$\frac{x^3 - 12x^2 - 42}{x-3} = \underbrace{x^2 - 9x - 27}_{q(x)} \underbrace{- \frac{123}{x-3}}_{r(x)/g(x)}$$

The long division algorithm for arithmetic can be viewed as a special case of the above algorithm, in which the variable x is replaced by the specific number 10.