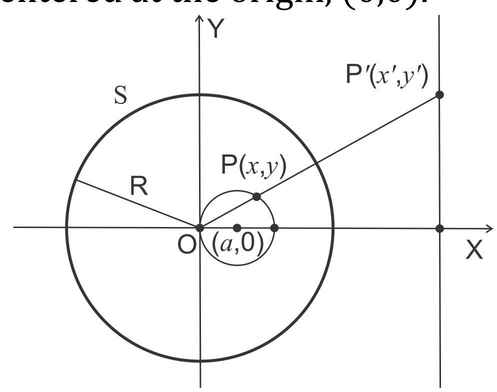
Homework due March 7, 2021.

Problems.

1. Given circle C and its image C' of find the inversion circle, S, which transforms one into another. Consider three cases:
   a. circles C and C' are crossing, i.e. have two common points
   b. circles C and C' are touching, i.e. have one common point
   c. circles C and C' have no common points
2. Find the distance between two parallel straight lines that are images of the two circles with the radii r_1 and r_2, which are tangent at the center O of the inversion circle S with radius R.
3. Express the similarity coefficient between circle L and its image L' through radius of the inversion circle R and length of the tangent, $|OT|$. What happens if $|OT| = R$?
4. Consider inversion with respect to circle S centered at the origin, (0,0). Image of point $P(x, y)$ is point $P'(x', y')$. Prove that the transformation of coordinates is (see figure),

$$x' = x\frac{R^2}{x^2 + y^2}$$
$$y' = y\frac{R^2}{x^2 + y^2}$$



5. Prove that given any two circles, there is some third circle such that the first two circles are images of each other under inversion through the third circle.
6. Let g(n) be a function that counts multiples of 2: for all n, g(n) is the number of even positive integers in [0,n] (including 0 and n). Let h(n) be defined as h(n)=n-g(n). Construct a function f such that f(0)=1 and for all positive integers n, we have h(n+1)=h(n)+1.
7. What would happen in problem 6 if g(n) counts both multiples of 2 and multiples of 3 in [0,n]? Can you tell what the range of f will be?