MATH 9: WINTER BREAK

2020 DEC 20

Winter Challenge Problems

1. Describe necessary and sufficient conditions that a hexagon's angles must fulfill in order for the hexagon to be cyclic. Next, suppose you are given a cyclic hexagon whose opposite sides are parallel; prove that the three line segments connecting the midpoints of opposite sides meet at a point.

2. Call a set S of natural numbers *symmetric* if $\{(M+m-s) \big| s \in S\}$ is equal to S, where M and m are the maximum and minimum element of S. How many subsets of $\{0,1,2,3,4,5,6,7,8,9\}$ are symmetric?

3. Suppose A and B are sets, and f is a function from A to B that is injective, and g is a function from B to A that is injective. Prove that there exists a bijective function from A to B.

As a reminder, given a subset of $X \times Y$, this subset is said to be a *function from X to Y* if it is left-total and right-definite; *injective*, if it is left-definite; *bijective*, if it is right-total and left-definite.